

**M.Sc. Examination, 2018**  
**Semester-II**  
**Statistics**  
**Course : MSC-21**  
**(Inference-II)**

**Time : 3 Hours**

**Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. (a) State and prove Neyman-Pearson fundamental lemma.  
(b) Find an Mp size  $\alpha$  test of  $H_0 : X \sim f(x) = 4x$  if  $0 < x < \frac{1}{2}$  and  $= 4-4x$  if  $\frac{1}{2} \leq x < 1$ ,  
against  $H_1 : X \sim f(x) = 1$  if  $0 < x < 1$ . 6+4=10
2. (a) Prove that if a family has  $\{f_\theta : \theta \in \Theta\}$  Has MLR in statistic. T, then any one sided  
test based on T has monotone power function while testing a one sided hypothesis.  
(b) Show that Cauchy  $(0, \sigma)$  possesses MLR but Cauchy  $(\theta, 0)$  does not have it. 6+4=10
3. (a) Define a  $\alpha$ -similar test.  
(b) Stating the necessary condition, prove that a  $\alpha$ -similar test is unbiased always and  
vice versa.  
(c) Consider the problem of testing  $H_0 : \theta = 1$  vs.  $H_1 : \theta = \frac{1}{2}$  where  $\theta$  is mean of  
Poisson variable. Let X and Y be a random sample from Poisson ( $\theta$ ) distribution.  
Consider the test process;  
Reject  $H_0$  if either  $X = 0$  or ( $X = 1$  and  $X + Y \leq 2$ ) otherwise accept  $H_0$ .  
Show that the power of the test is  $\frac{3}{4}e^{-1} + e^{-1/2}$ . 2+5+3=10
4. (a) Find the mean and variance of simple linear rank statistic.  
(b) Show that one sample kolmogorov-simimnov test statistic is exactly distribution  
free. 5+5=10
5. (a) For a uniparametric exponential family  $\{f_x(\theta), \theta \in \Theta\}$ , to test  $H_0 : \theta = \theta_0$  against  
 $H_1 : \theta \neq \theta_0$ ; establish the conditions on the basis of which an UMP unbiased test  
would exist. 6  
(b) Under two sample location problem, suggest a test statistic based on rank of the 2<sup>nd</sup>  
sample. What is the large sample distribution of it? 4
6. (a) Define Mann-whitney U statistic. Deduce its mean and variance.  
(b) Derive the null distribution of Wald-Wolfwitz run test statistic. 6+4=10