

M.Sc. Examination 2018
Semester-II
Computer Science
Course : MCSC-21
(Design and Analysis of Algorithms)

Time : 3 Hours

Full Marks : 40

Questions are of value as indicated in the margin.

Answer Question No. 1 and **any four** from the rest.

1. a) Explain asymptotic tight bound, asymptotic upper bound and asymptotic lower bound.
- b) Prove that $n^2 - 5n + 6 = \Theta(n^2)$.
- c) Compute the time complexity of the following program pseudocode:

```
A()
{
    int p, k, n;
    scan (p, k);
    n = pk;
    for (i = 1; i ≤ n; i = i * p)
    {
        j = p;
        while (j ≤ n)
            j = jp;
    }
}
```

- b) Can Master method be applied to solve the recurrence

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \lg n? \text{ Why or why not?} \quad [3 + 2 + 1\frac{1}{2} + 1\frac{1}{2} = 8]$$

2. a) What is Amortized analysis? Explain with an example.
- b) Show that if $f(n) = \Theta(n^{\log_b a} \lg^k n)$ where $k \geq 0$, then the master recurrence $[T(n) = aT\left(\frac{n}{b}\right) + f(n)$ where $a \geq 1$ and $b > 1$ be constants and $f(n)$ be a function] has solution $T(n) = \Theta(n^{\log_b a} \lg^{k+1} n)$. Assume that n is an exact power of b . Perform all the steps from the beginning. [3+5=8]
3. a) Explain Divide and Conquer method of problem solving and describe the time complexity in the recurrence form.
- b) How can it be used to solve Integer Multiplication problem or Matrix Multiplication problem?

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- c) By using the established recurrence relation, find the time complexity of the above problem solved by you. [(2+1)+4+1=8]
4. a) With an example explain the Bellman-Ford algorithm (let there be at least 5 nodes and 10 edges).
- b) How does it differ from Dijkstra's algorithm?
- c) Multiply two matrices $(1 \ 2 \ 3)_{1 \times 3}$ and $(5 \ 6 \ 7)_{3 \times 1}^T$ by Strassen's algorithm. [3+1+4=8]
5. a) Write an algorithm to multiply two complex numbers $a + ib$ and $c + id$ by using only three real multiplications. The algorithm should take a, b, c and d as input and produce that real component $ac - bd$ and the imaginary component $ad + bc$ separately.
- b) Show that a full parenthesization of an n -element expression has exactly $n - 1$ pairs of parentheses.
- c) What is complexity class? Explain different complexity classes. [2+2+4=8]
6. a) Prove that a directed graph G is acyclic if and only if a depth-first search of G yields no back edges.
- b) Write an algorithm to establish all the paths between any two vertices of a graph. Analyse its time complexity.
- c) Write an algorithm to find all strongly connected components of a graph. Illustrate with an example. [2+3+3=8]
7. a) Write matrix inversion algorithm.
- b) Solve the following system of linear equations by matrix inversion method:
- $$x + 2y = 3$$
- $$3x + 4y + 4z = 7$$
- $$5x + 6y + 3z = 8$$
- [4+4=8]
-