B.A. (Honours) Examination, 2018

Semester-II (CBCS)

Economics

Paper-CC-4 (Core)

(Mathematical Methods for Economics-II)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin

UNIT-I

Answer ***any two*** questions

1. (a) Let A and B be two invertible matrices with same dimensions. Show that AB is also invertible and $(AB)^{-1} = B^{-1}A^{-1}$

(b) Find the inverse of $C = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$

(c) Find a 2×2 matrix A such that A^2 is diagonal but not A.

(d) Using the property of invertibility, find the inverse of $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$

(e) Find x, y and z such that A is symmetric; where $A = \begin{pmatrix} 2 & x & 3 \\ 5 & 4 & y \\ z & 1 & 7 \end{pmatrix}$

2+1+3+6+3=15

2 (a). For what values of x the matrix A is singular? Where $A = \begin{pmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{pmatrix}$

(b) Express the matrix $A = \begin{pmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{pmatrix}$ as the sum of two matrices of which one is symmetrical and the other is skew-symmetrical

(c) Solve the following system of equations by matrix method:

$$x + 2y - z = -9$$
$$2x - y + 3z = -2$$
$$3x + 2y + 3z = 9$$

(d) Compute the Adjoint of $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}$ 3+3+6+3=15

3. (a) Define sub-matrix and minor

(b) Define rank of a matrix

(c) Find the rank of a matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{pmatrix}$

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(d) Let B = $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 8 & 9 \end{pmatrix}$. Find $|B|$. 3+2+6+4=15

4. (a) Find the values of the following determinants without expanding:

(i) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$

(b) Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$

(c) Solve the following system of equations using determinants:

$3y + 2x = z + 1$
$3x + 2z = 8 - 5y$
$3z - 1 = x - 2y$ 3+3+5+4=15

UNIT-II

Answer ***any two*** questions

5. (a) From the first principles (using limits), find f_x, f_y *and* f_{xy} *if* $f(x,y) = 2x^2 + 3xy$

(b) If $V = ax^2 + 2hxy + by^2$, then show that

$$V_x^2 V_{yy} - 2V_x V_y V_{xy} + V_y^2 V_{xx} = 8(ab - h^2)V$$ 8+7=15

6 (a) State Euler's Theorem

(b) Using Euler's Theorem, prove that:

(i) If $U = {}^{y}/_{z} + {}^{z}/_{x} + {}^{x}/_{y}$; then $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} + z\frac{\partial U}{\partial z} = 0$

(ii) If $V = x^2 + y^2 + z^2$; then $xV_x + yV_y + zV_z = 2V$ 3+6+6=15

7. Suppose the Utility function of a consumer consuming two commodities by x *and* y units is given by $U = 10x^{1/2}y^{1/2}$

(i) If he has money income M and prices P_x *and* P_y, derive his demand functions for x *and* y.

(ii) What is his optimum consumption of two commodities if money income is Rs. 120, $P_x = 2$ *and* $P_y = 6$?

(iii) What is the value of Lagrangian multiplier at this level of prices and income? What is the marginal utility of money in this situation? 8+3+4=15

8. (a) Show that the tangent at point (a,b) to the curve $(x/a)^3 + (y/b)^3 = 2$ *is* $\frac{x}{a} + \frac{y}{b} = 2$

(b) Find at what points on the curve $y = 2x^3 - 15x^2 + 34x - 20$ the tangents are parallel to $y + 2x = 0$.

(c) If $lx + my = 1$ is a normal to the parabola $y^2 = 4ax$, then show that $al^3 - 2alm^2 = m^2$ 5+5+5=15
