M.Sc. Examination, 2018
Semester-III
Computer Science
Course : MCSC-31
(Algorithmic Graph Theory)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer Question No. **1** and **any four** from the rest.

1. a) Prove that in a nonseparable graph the set of edges incident on each vertex of G is a cutest.

   b) Prove that Kuratowski's second graph is nonplanar.

   c) Prove that every connected graph with three or more vertices has at least two vertices which are not cut vertices.

   d) Explain chromatic partitioning and uniquely colorable graph. 4×2=8

2. a) Write an algorithm to check whether two graphs are 1-isomorphic or 2-isomorphic. Give one example of each to illustrate all the steps.

   b) Write an algorithm to detect the planarity of a disconnected graph. Consider an example to describe all the steps. 4+4=8

3. a) Prove that a self-loop free planar graph is 2-connected if any only if its dual is also 2-connected.

   b) Let G be a planar graph having n vertices, e edges, r regions and k components. Establish the relation among n,e,r and k. 4+4=8

4. Write an algorithm to find the chromatic polynomial of a disconnected graph. Illustrate this algorithm with an example having at least 3 components (non-isomorphic) and each component having at least 4 vertices. 4+4=8

5. a) Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length.

   b) Show that the chromatic polynomial of a graph consisting of a single circuit of length n is $P_n(\lambda) = (\lambda - 1)^n + (\lambda - 1)(-1)^n$. 4+4=8

6. a) Define perfect graph and $p-$critical graph.

   b) If G is a $p-$critical graph on n vertices, then prove that

      i) $\alpha(G)\omega(G) = n - 1$ and

      ii) $\forall$ vertices x *of* $G, \alpha(G) = k(G - x)$ and $\omega(G) = \chi(G - x)$. [Notations are as usual]

   c) Describe a procedure to recognize whether a graph is a triangulated or not.

   2+2+4=8

7. a) Prove that a permutation graph is a transitively orientable graph.

P.T.O.

b) Let $G = V, E)$ be a triangulated graph and let σ be a perfect elimination order for G. Prove that every maximal clique in G is of the form $X_u \cup \{u\}$ where $X_u = \{x \in Adj(u) : \sigma^{-1}(u) < \sigma^{-1}(x)\}$.

c) Describe a procedure to recognize a comparability graph. Illustrate with an example.

2+2+4=8
