B.Sc.(Honours) Examination, 2018

Semester–V

Physics (Honours)

Elective Course: BPE-1

(Group Theory and Tensor Analysis)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Symbols bear their usual meanings. Einstein Summation convention is used wherever applicable.

Answer ***any four*** questions

1. (a) What is a binary operation?

   (b) State the group axioms.

   (c) Write down the group multiplication table of the set $\mathscr{G} = \{1, i, -1, -i\}$. What figure do these elements create in Argand plane? 2+4+(2+2)

2. (a) Show that all the symmetry operations on any system form a group under the composition rule of ordinary multiplication.

   (b) A set of all non-zero number with unit modulus form a group under ordinary multiplication.

   (c) What is an Abelian group? Show that a set Q^+ of all positive numbers forms an Abelian group for the composition '$\circ$' defined as, $\forall\ a, b \in Q^+$,

$$a \circ b = \frac{ab}{2}$$

2+3+(2+3)

3. (a) State and prove the rearrangement lemma.

   (b) Prove that the identity element and the inverse of an element in a group are unique.

   (c) Prove that a non-empty subset H of a group $(\mathscr{G}, \circ)$ is a subgroup if

      i. $\forall\ a, b \in H \Longrightarrow a \circ b \in H$,

      ii. and $\forall\ a \in H \Longrightarrow a^{-1} \in H$.

   (d) Show that a coset is not a subgroup of a group $(\mathscr{G}, \circ)$. (1+2) + 2+3+2

4. (a) What is covariant divergence? Show that for an antisymmetric tensor $A^{\mu\nu}$

$$\nabla_\nu A^{\mu\nu} = \frac{1}{\sqrt{-g}}\partial_\nu \left(\sqrt{-g}\ A^{\mu\nu}\right)$$

   (b) Show that covariant derivatives do not commute under interchange of indices.

   (c) What is Einstein tensor? What is the mathematical reason for introducing such a tensor? (1+4)+3+(1+1)

5. (a) State and explain the equivalence principle.

   (b) Describe the general relativistic paradigm of gravitation as opposed to the Newtonian paradigm of the same.

   (c) Write down the Einstein equation for gravitational field clearly mentioning the unit system. (2+2)+4+2

6. (a) Find out the expression for Riemann tensor.
   (b) Show that
$$R_{\lambda\{\mu\nu\kappa\}} = 0.$$
   (c) Proof the Bianchi identity for the Riemann tensors. 3+4+3

7. (a) Show that the Ricci tensor is symmetric.
   (b) Find out the number of independent components of the Ricci tensor.
   (c) What are the properties of a geodesic?
   (d) Find out the geodesic equation. 2+2+2+4