B.Sc. (Honours) Examination, 2018

Semester–III (CBCS)

Physics (Core)

Core Course: CC-5

(Mathematical Physics-II)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Symbols bear their usual meanings.

Answer ***any four*** questions

1. a) A periodic function $f(x)$ is defined as $f(x)=x^2, -\pi \le x \le \pi$. Expand $f(x)$ in Fourier series.

   b) From (a) find out the sum of the series

   $$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots\ldots\ldots\ldots\ldots$$

   c) Establish the recursion relation

   $$\Gamma(n+1)=n\Gamma(n)$$

   d) State the normal law of errors. 4+2+2+2

2. a) Obtain the complete series solution of the Bessel's differential equation

   $$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-n^2\right)y=0$$

   where n is an integer.

   b) Can you expand $f(x)=\tan x$ in Fourier series? Explain. 8+2

3. a) Prove the following orthogonality relation for Legendre polynomials in the interval $-1 \le x \le 1$:

   $$\int_{-1}^{1} P_n(x)P_m(x)dx=0$$

   if $m \ne n$.

   b) Given $\int_0^\infty \frac{x^{n-1}}{1+x}dx=\frac{\pi}{\sin n\pi}$, show that

   $$\Gamma(n)\Gamma(1-n)=\frac{\pi}{\sin n\pi}$$

   c) Show that for an odd function, the Fourier series is a sine series. 5+3+2

4. a) Starting from the generating function for the Legendre polynomials

   $$\left(1-2xt+t^2\right)^{-\frac{1}{2}}=\sum_{n=0}^{\infty}P_n(x)t^n$$

   Show that $(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$.

   b) State and prove Parseval's identity for Fourier series.

   c) Evaluate $\int_0^{\pi/2}\sin^4\theta\cos^5\theta d\theta$. 3+4+3

5. a) Establish the following recurrence relations for Bessel function:

   (i) $xJ_n'(x)=nJ_n(x)-xJ_{n+1}(x)$

   (ii) $\frac{d}{dx}\left(x^{-n}J_n(x)\right)=-x^{-n}J_{n+1}(x)$

P.T.O.

b) Using the method of separation of variables, solve the following partial differential equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

with $u(0, t) = u(l, t) = 0$, $u(x, 0) = a \sin\frac{\pi x}{l}$ and $\frac{\partial u(x, 0)}{\partial t} = 0$. 5+5

6. a) From the given generating function

$$e^{x^2-(t-x)^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$$

Establish the Rodrigue's formula for Hermite polynomials.

b) Hence evaluate $H_3(x)$.

c) Solve the Laplace's equation in two dimensional polar form given by

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$ 4+2+4

7. a) Consider an elastic string of length *l* fixed at two ends under a constant tension *T*. Show that the transverse vibrations of the string leads to the one-dimensional wave equation.

b) Show that $\text{erf}(-x) = -\text{erf}(x)$.

c) Least-square fit the given data

x	2	4	6	8	10	12
y	2	4	4	5	5	7

Hence find the error on the slope and intercept of the fitted line. 3+2+(3+2)

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