

Undergraduate Examination, 2017
Semester-I (CBCS)
Physics
Generic Elective Course: GEC-1
(Mechanics)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.
Answer *any four* questions

1. a) Calculate the vector product of the vectors $\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{B} = 3\hat{i} + \hat{j} + \hat{k}$ and also show that the resulting vector is perpendicular to both \vec{A} and \vec{B} . 2
- b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$, where $r = \sqrt{x^2 + y^2 + z^2}$ (magnitude of \vec{r}). 2
- c) Show that $\vec{\nabla} \times (\phi \vec{A}) = (\vec{\nabla} \phi) \times \vec{A} + \phi (\vec{\nabla} \times \vec{A})$. 3
- d) Prove that $\vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3} \right) = 0$. 3

2. a) Solve the following differential equations:
 - i) $\frac{dy}{dx} + y = \sin x$
 - ii) $\frac{dy}{dx} - y = xy^5$
 - iii) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xe^x$ 2+3+3
- b) Determine whether the set $\{e^x, e^{-x}\}$ is linearly dependent on $(-\infty, \infty)$. 2

3. a) What is an inertial frame? Write down the Galilean transformation equations and show that under the Galilean transformation, Newton's second law remains form-invariant. 1+2
- b) Show that for a system of particles, the torque about any point due to mutually interacting forces, assumed to be central, vanishes. Hence show that for the above system, $d\vec{L}/dt = \vec{N}$ (usual notation). State, from above, the law of conservation of angular momentum. 1+2+1
- c) Discuss on the total energy of a system of particles under conservative forces. 3

4. a) Deduce the height of a geostationary satellite above the surface of the earth so that the satellite can be placed in its parking orbit. [Consider the radius of earth to be 6400 km] 5
- b) The earth's mass is 80 times that of the moon. The diameters of the earth and the moon are 12,800 km and 3200 km respectively. What is the value of "g" on the moon? 2
- c) Write down the Kepler's laws of planetary motion. Comment on the orbital velocity of the planets on the basis of Kepler's laws. 2+1

5. a) Show that the total energy of a particle executing Simple Harmonic Motion (SHM) is a conserved quantity. 6
- b) The amplitude of a particle executing SHM is 5 m and its acceleration at a distance of 2 m from the mean position is 8 m/s². Find the velocity of the particle when it is at a distance of 3 m from the mean position. 2
- c) Calculate the ratio of kinetic energy of a particle under SHM at displacement one fourth to one third of the amplitude. 2

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6. a) What do you mean by "elastic limit"? State Hooke's law. 1+1
b) Calculate the total amount of work done per unit volume in stretching the wire through a certain length " l ". 3
c) Prove that for a homogeneous isotropic medium, $Y = 3K(1 - 2\sigma)$. The symbols have their usual significance. 5
7. a) Deduce the standard expression for the torsional couple per unit twist of a solid cylinder. 5
b) Compare the torsional rigidity between a solid cylinder and a hollow cylinder. *You have to consider that the two cylinders are made up of same material; they have the same length and mass.* 3
c) A wire is found to cross its elastic limit when the strain exceeds $1/100$. It has a diameter of 0.04 cm and Young's modulus for its material is 1.2×10^{11} N/m². What is the highest mass that can be hung from the free end of the wire without causing permanent stretching? 2
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