B.Sc.(Honours)Examination, 2018

Semester–I (CBCS)

Physics (Core)

Core Course: CC-1

(MathematicalPhysics-I)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer***anyfour*** questions

1. a) Prove that a Separable equation is always exact. 1

   b) Solve $\frac{dy}{dx} + 2xy = x^3 + x$ if $y = 2$ when $x = 0$. 3

   c) Solve the following differential equations:

   i) $\left(3x^2 + y\cos x\right)dx + \left(\sin x - 4y^3\right)dy = 0$

   ii) $\frac{dy}{dx} = -\frac{2}{y} - \frac{3y}{2x}$ 3+3

2. a) Explain what is Wronskian and discuss its role in the general solution of a second order differential equation. 3

   b) Obtain a general solution of the following differential equation:

   i) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = x^2 + 2e^{3x}$

   ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = xe^{-x}$ 4+3

3. a) If $\vec{A} = 2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}$ and $\vec{B} = x^2\hat{i} + yz\hat{j} - xy\hat{k}$, find $\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A}$. 2

   b) Find $\vec{\nabla}\phi$ if $\phi = ln\left|\vec{r}\right|$. 2

   c) Prove that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = C$, where C is a constant. 2

   d) Prove the vector identity:

   $$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right) - \vec{\nabla}^2\vec{A}.$$ 4

4. a) If $\vec{A} = \left(2xy + z^3\right)\hat{i} + \left(x^2 + 2y\right)\hat{j} + \left(3xz^2 - z\right)\hat{k}$, find a Scalar function ϕ such that $\vec{A} = \vec{\nabla}\phi$. 3

   b) Evaluate $\vec{\nabla}\cdot\left[r\vec{\nabla}\left(\frac{1}{r^3}\right)\right]$. 3

   c) If $\vec{A} = (2x - y + 4)\hat{i} + (5y + 3x - 6)\hat{j}$, evaluate $\oint \vec{A}\cdot d\vec{r}$ around a triangle with vertices at (0, 0, 0), (3, 0, 0), (3, 2, 0). 4

5. a) If $\vec{A} = \left(2x^2 - 3z\right)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, evaluate $\iiint_V \vec{\nabla}\cdot\vec{A}\,dV$ where V is the closed region bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $2x + 2y + z = 4$. 4

   b) Write down Green's theorem in plane. Using this evaluate the integral,

   $$\oint_C \left[\left(x^2 + xy\right)dx + \left(x^2 + y^2\right)dy\right],$$

   where$C$ is the Square formed by the line $y = \pm 1,\ x = \pm 1$. 1+3

P.T.O.

c) Find expression for the elements of area in orthogonal curvilinear coordinates. 2

6. a) Obtain the expression for $\vec{\nabla} \times \vec{A}$ in orthogonal curvilinear coordinates. 4

b) Paraboloidal coordinates u, v, φ are defined in terms of Cartesian coordinates by

$$x = uv\cos\varphi,\ y = uv\sin\varphi,\ z = \frac{1}{2}\left(u^2 - v^2\right)$$

where $u \geq 0,\ v \geq 0,\ 0 \leq \varphi < 2\pi$. Show that the system of coordinate is an orthogonal one and determine its scale factors. Write down the u-component of $\vec{\nabla} \times \vec{a}$. 4

c) Express the velocity $\vec{V}$ of a particle in cylindrical coordinates. 2

7. a) What is the 'addition rule of Probability' of mutually exclusive events? 2

b) Prove the 'multiplication law of Probability'

$$P(A \cap B) = P(A)P(B).$$ 3

c) What is Poisson's distribution? Find an expression for it. Calculate the mean of this distribution. 1+2+2