B.Sc.(Honours)Examination, 2018

Semester–I

Physics (Honours)

Course: BPC-11

(Mathematical Methods-1)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer***anyfour*** questions

1. a) Find a unit normal to the surface $2x^2+4yz-5z^2=-10$ at the point $p(3,-1,2)$. 2

   b) If $\phi = x^2yz^3$ and $\vec{A}=xz\hat{i}-y^2\hat{j}+2x^2y\hat{k}$, find

   (i) $\vec{\nabla}\phi$ (ii) $\vec{\nabla}\times\vec{A}$ (iii) $\vec{\nabla}\cdot(\phi\vec{A})$ (iv) $\vec{\nabla}\times(\phi\vec{A})$ 5

   c) If $\vec{f}=x\hat{i}+y\hat{j}+z\hat{k}$, then prove that $\vec{\nabla}r^n=nr^{n-2}\vec{r}$, where $r=\sqrt{x^2+y^2+z^2}$. 3

2. a) Prove that $\vec{\nabla}\times(\phi\vec{A})=\phi(\vec{\nabla}\times\vec{A})+(\vec{\nabla}\phi)\times\vec{A}$. 4

   b) Evaluate $\vec{\nabla}\cdot(r^3\vec{r})$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. 3

   c) Calculate the work done when a force $\vec{F}=3xy\hat{i}-y^2\hat{j}$ moves a particle in the *xy* plane from (0, 0) to (1, 2) along the parabola $y=2x^2$. 3

3. a) If $\vec{A}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$, evaluate $\iint_S \vec{A}\cdot\hat{n}\,dS$ where *S* is the surface of the cube bounded by $x=0,\ x=1,\ y=0,\ y=1,\ z=0,\ z=1$. 4

   b) Express Green's theorem in the plane in vector notation. 3

   c) Find the square of the element of arc length (dS^2) in Cylindrical coordinates. 3

4. a) Obtain the expression for $\vec{\nabla}\cdot\vec{A}$ in orthogonal curvilinear coodinates. 4

   b) Find the volume element in spherical coordinates. 2

   c) If u_1, u_2, u_3 are orthogonal curvilinear coordinates, calculate the Jacobian of *x*, *y*, *z* with respect to u_1, u_2, u_3. 4

5. a) What is a Fourier series? Can you expand $f(x)=\tan x$ in a Fourier series? Explain. 1+1

   b) Calculate the value of a_n in the Fourier series of $f(x)=|x|$ in the interval $(-\pi,\pi)$. 2

   c) Find the Fourier series for the following function

   $$f(x)=\begin{cases}0 \text{ for } -\pi\le x<0\\ x \text{ for } \ 0\le x<\pi\end{cases}$$

   Hence show that $1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$. 4+2

6. a) Determine whether the differential equation $y'=y/x$ is exact. 2

   b) Solve the following differential equations:

   (i) $\frac{dy}{dx}+y=y^2e^x$ (ii) $\frac{d^2y}{dx^2}+w^2y=0$ (iii) $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$ 3+2+3

P.T.O.

7. a) Solve the equation $\frac{d^2\vec{r}}{dt^2} = -g\hat{k},$ where g is a constant. Given that $\vec{r} = 0, \frac{d\vec{r}}{dt} = v_0\hat{k}$ at $t = 0.$ 2

b) Obtain a general solution of the following differential equations:

i) $\frac{d^2y}{dx^2} + y = e^x \cos x$

ii) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = x^4e^{2x}$ 4+4