**M.Sc. Examination, 2018**
**Semester-III**
**Statistics**
**Course: MSC-31**
**(Real Analysis and Measure Theory)**

**Time : 3 Hours** **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. Let $\{A_n\}$ be a sequence of subsets of $\Omega$. Find the $\limsup\{A_n\}$ and $\liminf\{A_n\}$ for the following
i) $A_n = [0, \frac{n}{n+1})$ ii) $A_n = [0, 1-\frac{1}{n}]$ iii) $A_n = [0, 1+\frac{1}{n})$ iv) $A_n = C$ if $n$ is odd, $A_n = D$ if $n$ is even. 10

2. State and prove monotone convergence theorem. Hence or otherwise prove Fatou's lemma. 10

3. If $f$ and $g$ are two integrable functions, show that $f \pm g$ is also integrable and $\int(f \pm g)d\mu = \int f d\mu \pm \int g d\mu$. 10

4. State and prove the continuity theorem of measure. 10

5 (a) Prove that the union of a finite number of closed sets is closed. Does this hold for an arbitrary number of closed sets.

(b) Exhibit an open cover of the set $\{\frac{1}{n} : n \in \mathbf{N}\}$ that has no finite subcover. Is the set compact? 5+5

6. (a) Let $f : S \to \boldsymbol{R}$ be continuous on a compact set $S$. Prove that $f(S)$ is a compact set.
(b)Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+3)^n}{n+1}$ 5+5

7. (a) Let $\{f_n\}$ be a sequence of functions defined on a set $E$. Prove that $\{f_n\}$ converges uniformly to $f$ on $E$ if and only if $\sup_{x \in E} |f_n(x) - f(x)| \to 0$ as $n \to \infty$.

(b) Show that $\sum_{n=1}^{\infty} \frac{x}{n + n^2x^2}$ is uniformly convergent for all real values of $x$. 5+5