M.Sc. Examination, 2018
Semester-I
Statistics
Course : MSC-14

(Statistical Inference-1)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Define a minimal sufficient statistic. If $X_1, X_2, ...X_m$ are distributed as $N(\mu, \sigma_1^2)$ and $X_{m+1}, X_{m+2}, ..., X_{m+n}$ are distributed as $N(\mu, \sigma_2^2)$ independently, obtain a minimal sufficient statistic for $(\mu, \sigma_1^2, \sigma_2^2)$.

   b) State and prove the Rao-Blackwell theorem. 6+4=10

2. a) Let U_g and U_0 be, respectively, the class of all unbiased estimators of $\gamma(\theta)$ with finite variances and the class of all unbiased estimators of zero with finite variances. Then show that T is an uniformly minimum variance unbiased estimator (UMVUE) of $\gamma(\theta)$ if and only if cov(T, h) = 0 $\forall \theta \in \Omega, \forall h \in U_0$.

   b) Let the probability distribution of a random variable X be

   $P_\theta[X = -1] = \theta, P_0[X = x] = (1-\theta)^2 \theta^x, x = 0,1,2,...$ Show that any estimable function $\gamma(\theta)$ admits an UMVUE if and only if $\gamma(\theta) = \alpha + b(1-\theta)^2$ for some constants a, b. Also mention the UMVUE of $\gamma(\theta)$. 5+5=10

3. a) Describe Maximum likelihood method for parameter estimation. State its properties.

   b) Let $(X_1, X_2, \ldots, X_n)$ be a random sample from $R(\theta, \theta+1)$ with unknown parameter θ. Find the maximum likelihood estimator of θ. 6+4=10

4. a) Consider the exponential family of distributions

   $$P = \{f_\theta(x); \theta \in \Omega\},$$

   where $f_\theta(x) = k\theta e^{Q(\theta)t(x)} h(x)$.

   Prove that if P is of the above form, then T=t(x) is complete sufficient.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $R(\theta_1, \theta_2)$. Is $T = (X_{(1)}, X_{(n)})$ sufficient for $\theta = (\theta_1, \theta_2)$? Is it complete? Give reasons for your answers. 4+6=10

5. a) Find the expression for variance of U_n, a U-statistic based on n iid observations and a symmetric Kernel of size m (<n). also find the limit of [n Var (U_n)] as $n \to \infty$.

   b) State the result on asymptotic normality of $U_{n'}$. 8+2=10

6. a) Derive Bayes estimate of a real parametric function $\gamma(\theta)$ under squared error loss.

P.T.O.

b) suppose $X_i; i = 1(1)n$ follows Bernoulli with parameter θ . The prior distribution of θ is Beta with parameters α and β. Find Bayes estimate of θ under squared error loss. 4+6=10

7. Write short notes on **any two** of the following : 5+5=10

a) Bhattacharya's lower bound and its use

b) Minimum Chi-square method of estimation

c) U-statistic and Kendall's τ

d) Loss functions in Bayesian estimation
