M.Sc. Examination, 2018
Semester-I
Statistics
Course : MSC-13

(Stochastic Process and Distribution Theory)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Group-A (Stochastic Process)

Answer **any two** questions

1. a) Define covariance stationarity of a stochastic process. Cite an example where covariance stationarity does not imply strict stationarity.

   b) Let $\{y_n, n \geq 1\}$ be a sequence of independent random variables with $P(Y_n=1) = p = 1 - P(Y_n = -1)$. Let X_n be such that $X_0 = 0$

   $$X_{n+1} = X_n + Y_{n+1}.$$

   Examine if $\{X_n, n \geq 1\}$ is a Markov process.

   c) "If a markov Chain has limiting distribution it has stationary distribution as well" – Is the statement correct? Discuss with a suitable example. 3+4+3=10

2. a) What do you mean by an ergodic state? Suppose a Markov chain with state space S={0,1,2} has transition probability matrix.

   $$\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$$

   Check if the states are ergodic.

   b) Prove that in an irreducible Markov chain, all the states are of the same type. Then discuss on the states of a finite irreducible Markov Chain. (2+3)+(3+2)=10

3. a) Consider a gambler who at each play of the game either wins one unit of money with probability p or loses one unit of money with probability 1 – p. Let the game continues until the gambler's capital increases to a rupees or he goes broke. Find out the probability of his losing if at some point he has i unit of money.

   b) Consider a Poisson process $\{X(t), t \geq 0\}$. Let S_n denote the time of n occurrences of the event. Find the distribution of S_n. 6+4=10

4. a) Suppose illegal immigration to India occurs at a Poisson rate of 8 per week (say). The probability that an illegal immigrant is Bengali speaking is 1/6. Find the probability that no Bengali-speaking illegal immigrant will arrive in India during a period of 14 days. 5+5=10

b) In the context of continuous time Markov Chain establish Feller-Kolnogorov baekward and forward equations. 5+5=10

Group – B (Distribution Theory)

Answer **any two** questions

5. a) Derive the expression of the MGF of non-central χ^2 distribution with d.f n and non-centrality parameter λ.

   b) Prove that $P(x_n^2(\lambda) \le x) = P(X_1 - X_2 \ge \frac{n}{2})$, where $X_1 \sim$ Poisson $\left(\frac{x}{2}\right)$, $X_2 \sim$ Poisson $\left(\frac{\lambda}{2}\right)$, independently. 5+5=10

6. In the Gauss Markov setup, prove that the error sum of squares and sum of squares due to regression are independent x^2 random variables with degrees of freedom n-r and r respectively, r being the rank of the n × p design matrix X. You have to prove the necessary results. 10

7. Prove that the sample mean vector and dispersion matrix based on a normal data matrix distributed are independently distributed. Also find the sampling distribution of the sample dispersion matrix. 4+6=10

8. a) Find the sampling distribution of the sample Mahalanobis distance.

   b) Let $\underset{\sim}{X}_1, \underset{\sim}{X}_2 ... \underset{\sim}{X}_n$ $\underset{\sim}{i}.i.d$ $N_p(\underset{\sim}{\mu}, \Sigma)$. Derive the likelihood ratio test for testing $H_0 : R\underset{\sim}{\mu} = \underset{\sim}{r}$ with known R (matrix) and $\underset{\sim}{r}$ (vector). 5+5=10