M.Sc. Examination, 2018
Semester-I
Statistics
Course : MSC-11

(Linear Algebra and Linear Models)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Let A be an $m \times n$ matrix and let B be an $n \times m$ matrix, with $n \geq m$. Then show that the n eigenvalues of BA are m eigenvalues of AB with the extra eigenvalues being 0.

   b) Prove that if A is a symmetric matrix, then the eigenvalues of A are all real.

   c) Let A be a symmetric $n \times n$ matrix, and let $\lambda \neq \mu$ be eigenvalues of A with **x, y** as corresponding eigenvectors respectively. Then show that $\mathbf{x}'\mathbf{y}=\mathbf{0}$. 5+3+2=10

2. a) State and prove spectral decomposition theorem.

   b) Show that for any square matrix, the determinant is the product of all the eigenvalues and trace is the sum of all the eigenvalues. 5+5=10

3. a) State and prove a necessary condition for a linear parametric function $\lambda'\beta$ to be estimable under the Gauss-Markov linear model set up.

   b) Consider the model

$$E(y_1) = \beta_1 + 2\beta_2$$
$$E(y_2) = 2\beta_2$$
$$E(y_3) = \beta_1 + \beta_2,$$

   Find the residual sum of squares subject to the restriction $\beta_1 = \beta_2$. 5+5=10

4. a) With reference to the linear model

$$E(y_1) = \beta_1 + \beta_2,$$
$$E(y_2) = 2\beta_1 - \beta_2,$$
$$E(y_3) = \beta_1 - \beta_2,$$

   Where y_1, y_2, y_3 are uncorrelated with a common variance, answer the following questions:

   i. Find two different linear function of y_1, y_2, y_3 that are unbiased for β_1. Determine their variances and the covariance between the two.

   ii. Find two linear functions that are both unbiased for β_2 and are uncorrelated.

   iii. Write down the model in terms of the new parameters $\theta_1 = \beta_1 + 2\beta_2$, $\theta_2 = \beta_1 - 2\beta_2$.

b) Consider a linear model E(Y) = X β, where Y is the vector of *n* observations and β is the vector of *n* parameters. The (*i, j*)th element of the design matrix is of the form $1+ij(j+1), 1 \le i, j \le n$. Find the condition on *n* so that β will be estimable.

5. Consider the model

$$y_1 = \mu + \alpha_1 + \beta_1 + \in_1$$
$$y_2 = \mu + \alpha_1 + \beta_2 + \in_2$$
$$y_3 = \mu + \alpha_2 + \beta_1 + \in_3$$
$$y_4 = \mu + \alpha_2 + \beta_2 + \in_4$$
$$y_5 = \mu + \alpha_3 + \beta_1 + \in_5$$
$$y_6 = \mu + \alpha_3 + \beta_2 + \in_6$$

Answer the following questions with justification.

a) When is $\lambda_0\mu + \lambda_1\alpha_1 + \lambda_2\alpha_2 + \lambda_3\alpha_3 + \lambda_4\beta_1 + \lambda_5\beta_2$ estimable?

b) Is $\alpha_1 + \alpha_2$ estimable?

c) Is β_1 / β_2 estimable

d) Is $\mu + \alpha_1$ estimable?

e) Is $\alpha_1 - 2\alpha_2 + \alpha_3$ estimable?

f) What is the covariance between the BLUEs of $\beta_1 - \beta_2$ and $\alpha_1 - \alpha_2$, if they are estimable.

g) Obtain any linear function of observations belonging to the error space.

h) What is the rank of the estimation space?

6. In case of multiple linear regression equation, discuss the testing of $H_0 : \beta_1 = ... = \beta_p = 0$. Express the test statistic in terms of the multiple correlation.

10