B.Sc. (Honours) Examination, 2018
Semester-V
Statistics
Course : BSC-52
(Multivariate Analysis and Large Sample-II)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Show that $r^2_{1.23..p}$ may be regarded as the proportion of the predicted variance of x_1 that is explained by its linear regression on $x_2, x_2, \ldots, x_p$. to the total variance of x_1.

   b) Let $r_{1(2.34...p)}$ be the correlation between x_1 and the residual of x_2 remaining the regression of $x_3, x_4, \ldots, x_p$. Show that 5+5=10

   $$r^2_{1(2.34..p)} \leq r^2_{12.34..p}$$

2. a) In a multinomial distribution (n, $p_1, p_2, \ldots, p_{k-1}$) show that variance covariance matrix is nonsingular. Hence calculate $\rho_{1.23..\overline{K-1}}$.

   b) Let $\underline{X}$ be a p×1 vector whose probability density function is as follows.

   $f(\underline{x}) = Ke^{-1/2(\underline{x}-\underline{b})' A(\underline{x}-\underline{b})}$ where $\underline{b}$ is a scalar, A be a positive definite matrix and K is the normalizing constant.

   Determine K, $\underline{b}$ and A in terms of $E(\underline{X}) = \underline{\mu}$ and $V(\underline{X}) = \Sigma_{k \times K}$. 6+4=10

3. a) Each of the random variables X, Y, Z has mean zero and variance 1 while aX + by + cZ = 0. Find the dispersion matrix of X, Y, Z.

   Hence show that $a^4 + b^4 + c^4 \leq 2(b^2c^2 + c^2a^2 + a^2b^2)$.

   b) The random variables $X_1, X_2, \ldots, X_p$ have the Dirichlet distribution with p.d.f.

   $f(x_1, x_2, .., x_p) = c\, x_1^{\alpha-1} x_2^{\alpha-1} \cdots x_p^{\alpha-1}(1 - x_1 - .. - x_p)^{\alpha-1}$

   i) Find the marginal distribution of X_i

   ii) Find C in terms of α's

   iii) Find $E(X_i)$

   iv) Find $V(X_i)$

   v) Find cov (X_i, X_j) 4+6=10

4. a) Let $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$, split $\underline{X}$ as $\begin{pmatrix} \underline{X}_{(1)_{q\times1}} \\ \underline{X}_{(2)\overline{p-q}\times1} \end{pmatrix}$. Show that regression equation of $\underline{X}_{(1)}$ on $\underline{X}_{(2)}$ is linear.

P.T.O.

b) Let $\underline{X} \sim N_3(\underline{0}, \Sigma), \Sigma = \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{pmatrix}$

Show that for any c > 0

$P_r[(X_2^2 + c)\rho^2 - 2\rho(X_1X_2 + X_2X_3) + (X_1^2 + X_2^2 + X_3^2 - c) \leq 0]$

$= \int_0^c \frac{1}{2\pi} e^{-y/2} y^{1/2} dy$ 5+5=10

5. a) Show that $\rho_{1j} = 0 (j = 2,3,.., p) \Rightarrow \rho_{1.23..p} = 0$. Is the converse also true?

b) Suppose a population consists of k mutually exclusive classes, the proportion of members falling in the ith class being p_i, i = 1,2,..,k. In choosing of a s ample of size n, f_i turns out to be the frequency of sample unit into the ith class.

Deduce the large sample distribution of $\sum_{i=1}^{k} \frac{(f_i - np_i)^2}{np_i}$. 5+5=10

6. a) For a bivariate distribution, establish the equation of concentration of ellipse clearly mentioning all steps.

b) Write a short note on test of homogeneity of *l* populations with k classes each. 6+4=10

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