B.Sc. (Honours) Examination, 2018
Semester-V
Statistics
Course: BSC-51
(Testing of Hypotheses)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Distinguish between simple and composite hypotheses with illustration. 4
   b) Explain the concept of Type II error and Power curve. Let X have a Binomial distribution with n=10 and p where p ϵ (0.5, 0.25). If the observed value of X_1, a random sample of size one, is less than or equal to 3, we reject H_0=0.5, and accept H_1=0.25. Find the power function of the test. 6

2. a) Define UMP test. Let $x_1, x_2, \ldots x_n$ be random sample of size n from the distribution U (0,θ). Obtain an UMP test for testing $H_0: \theta \leq \theta_0$ against $H_1: \theta > \theta_1$. 5

   b) Let $H_0 : X \sim f_0(x) = 1,\ 0 < x < 1$ *and* $H_1 : X \sim f_1(x) = 2x,\ 0 < x < 1$. Based on a single observation obtain the most powerful test of size α =0.1 for testing H_0 against H_1. 5

3. a) Let $x_1, x_2, \ldots, x_n$ be independent random variables following N(θ,1) distribution. Develop LRT for testing θ=1 against θ≠1 3
   b) Let X be a random variable distributed in the interval (0, 1) with the density function

$$f(x,\theta) = \frac{\Gamma(3\theta)}{\Gamma(\theta)\Gamma(2\theta)} x^{\theta-1}(1-x)^{2\theta-1}$$

   Suppose we wish to test by means of the most powerful test, the following hypothesis based on a single observation of X: $H_0 = 1$ *against* $H_1 = 2$.
   Show that the critical region of the test has the following form for some constant k:

$$\{x \in (0,1) : \quad x(1-x)^2 > k\}$$ 3

   Suppose that X is observed to be 1/7.

   Using the shape of the critical region as obtained above in (b) obtain the most powerful test for testing H_0 against H_1. 3
   c) Hence, show that the p-value given by the above test is $\frac{27}{49}$. 1

P.T.O.

4. a) Let $x_1, x_2, ...x_n$ and $y_1, y_2, ...y_m$ be random samples from the distributions $N(\theta_1, \theta_3)$ and $N(\theta_2, \theta_4)$ respectively. Show that the likelihood ratio test for testing

$$H_0 : \theta_3 = \theta_4 \text{ , } \theta_1, \theta_2 \text{ unspecified}$$

against

$$H_1 : \theta_3 \neq \theta_4 \text{ , } \theta_1, \theta_2 \text{ unspecified}$$

can be based on the random variable

$$F = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2 \Big/ (n-1)}{\sum_{i=1}^{m}(y_i - \bar{y})^2 \Big/ (m-1)}$$ 5

b) Find the shortest length $100(1-\alpha)\%$ confidence interval for the parameter θ on the basis of a random sample $x_1, x_2, ...x_n$ from the distribution $U(0, \theta)$. 5

5. a) What do you understand by randomised test? Why it is necessary? Describe with illustration. 5

b) Let $x_1, x_2, ..., x_n$ be a random sample of size n from an exponential distribution with pdf

$$f_\theta(x) = \theta \exp(-\theta x), \quad x > 0$$

where $0 < \theta < \infty$. Derive the MP critical regions of size α for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ $(\theta_1 < \theta_0)$. Show that the test is actually UMP against one-sided alternative. 5

6. a) Define a most powerful test. A sample of size 1 is taken from the probability density function

$$f(x, \theta) = \frac{2(\theta - x)}{\theta^2}, \quad 0 < \theta < \theta.$$

Derive a size α most powerful test for testing $H_0 : \theta = \theta_0$ against $H_0 : \theta = \theta_1; \quad \theta_1 > \theta_0$. 6

b) Explain invariance and consistency of a test. 4