B.Sc. (Honours) Examination, 2018
Semester-I (CBCS)
Statistics
Course : CC-2
Calculus

Time : 3 Hours **Full Marks : 60**

Questions are of value as indicated in the margin

Answer any four questions

1. a. Give formal definition of limit of a function. Show that
$$i.\ \lim_{x\to x_0} x = x_0 \qquad ii.\ \lim_{x\to x_0} k = k, (k\ constant)$$
b. Prove that $\lim_{x\to 2} f(x) = 4$ *if*
$$f(x) = \begin{cases} x^2\ if\ x \neq 2 \\ 1\ if\ x = 2 \end{cases}$$
c. Give an example to show that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ do not exist but $\lim_{x\to a}[f(x) + g(x)]$ exists. 7+4+4=15

2. a. Define continuous extension to a point. Show that
$$f(x) = \frac{x^2 + x - 6}{x^2 - 4}$$
has a continuous extension to $x = 2$ and find that extension.

b. Show that if a function is derivable at any point, then the function is continuous at that point. Is the converse true? 6+5+4=15

3. a. State and prove Leibnitz theorem for successive derivation.
b. Find n-th order derivative for the following functions:
i. $\frac{1}{1-5x+6x^2}$ ii. $x^{n-1}\log x$ 7+8=15

4. a. State and prove the mean value theorem (MVT). Hence or otherwise show that functio with zero derivatives are constant.
b. Evaluate $\int_0^1 \int_x^1 e^{y^2} dy\, dx$ 9+6=15

5. a. State and prove the first fundamental theorem of calculus.
b. Find the area of the region between x-axis and the graph of
$$f(x) = x^3 - x^2 - 2x - 1 \qquad -1 \leq x \leq 2$$ 8+7=15

6. a. State and prove second fundamental theorem of calculus.
b. For $s > 0$, let
$\Gamma(s) = \int_0^\infty e^{-t}\, t^{s-1} dt$
Show that $\Gamma(s+1) = s\Gamma(s)$ and $\Gamma(n+1) = n!$ for any positive integer n.
c. Solve $\frac{dy}{dx} = \frac{x}{y}$ 8+4+3=15