**B.Sc. (Honours) Examination, 2018**
**Semester-III**
**Statistics**
**Course : BSC-31**
**Sampling Distribution**

**Time : 3 Hours** **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions from the following

1. (a) Show that the conditional distribution of $X$, for any given value of $X+Y$, is Hypergeometric in case $X$ and $Y$ are independent Binomial random variables with same probability of success.

   (b) Show that the conditional distribution of $X$, for any given value of $X+Y$, is Binomial in case $X$ and $Y$ are independent Poisson random variables. 5+5

2. (a) Let $X_1$ and $X_2$ be two independent Binomial random variables with parameters $(n_1, p)$ and $(n_2, q)$ respectively. Derive the sampling distribution of $X_1+n_2-X_2$.

   (b) Let $X$ be a continuous random variable having the *pdf* $f(x)$ and the *cdf* $F(x)$. Derive the sampling distribution of $Y = F(X)$. 5+5

3. Let $X_1, X_2, \cdots, X_n$ be a random sample of size $n$ drawn from a normal population with mean $\mu$ and variance $\sigma^2$. Show that the sample mean and variance are independently distributed. Hence or otherwise, derive the sampling distribution of the sample variance. 8+2

4. (a) Let $X_1$ and $X_2$ be a random sample of size two drawn from $R(0,1)$, where $R$ stands for rectangular population. Obtain the sampling distribution of $X_1+X_2$.

   (b) Let $X_1$ and $X_2$ be two independent random variables following Gamma distributions with parameters $(\alpha, p_1)$ and $(\alpha, p_2)$ respectively. Show that $U = X_1 + X_2$ and $V = X_1/(X_1 + X_2)$ are independently distributed. 5+5

5. Let $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$ be a random sample of size $n$ drawn from a Bivariate Normal population with parameters $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$. Derive the joint distribution of $(\overline{(X)}, \overline{(Y)})$. Also derive the distribution of the sample correlation coefficient when $\rho = 0$. 5+5

6. (a) Let the random variables $X$ and $Y$ follow jointly a Bivariate Normal distribution with $(0,0,1,1,\rho)$. Obtain the correlation coefficient between $G(X)$ and $H(Y)$, where $G(X)$ and $H(Y)$ are the *cfds* of $X$ and $Y$ respectively.

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(b) Let X
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(b) Let $X_1, X_2, \cdots, X_n$ be a random sample of size $n$ drawn from a population with the *cdf* $F(x)$. Derive the expectation of the $r$th order statistic. 5+5