B.Sc. (Honours) Examination, 2018
Semester-III
Statistics
Course : BSC-31
Sampling Distribution and Large Sampe Theory-I

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions from the following

1. (a) Show that the conditional distribution of X, for any given value of $X+Y$, is Hypergeometric in case X and Y are independent Binomial random variables with same probability of success.

   (b) Show that the conditional distribution of X, for any given value of $X+Y$, is Binomial in case X and Y are independent Poisson random variables. 5+5=10

2. (a) Let X_1 and X_2 be two independent Binomial random variables with parameters (n_1, p) and (n_2, q) respectively. Derive the sampling distribution of $X_1+n_2-X_2$.

   (b) Let X be a continuous random variable having the *pdf* $f(x)$ and the *cdf* $F(x)$. Derive the sampling distribution of $Y = F(X)$. 5+5=10

3. Let $X_1, X_2, \cdots, X_n$ be a random sample of size n drawn from a normal population with mean μ and variance σ^2. Show that the sample mean and variance are independently distributed. Hence or otherwise, derive the sampling distribution of the sample variance. 10

4. (a) Let X_1 and X_2 be a random sample of size two drawn from $R(0,1)$, where R stands for rectangular population. Obtain the sampling distribution of X_1+X_2.

   (b) Let X_1 and X_2 be two independent random variables following Gamma distributions with parameters (α, p_1) and (α, p_2) respectively. Show that $U = X_1 + X_2$ and $V = X_1/(X_1 + X_2)$ are independently distributed. 5+5=10

5. Let $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$ be a random sample of size n drawn from a Bivariate Normal population with parameters $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$. Derive the joint distribution of $(\overline{(X)}, \overline{(Y)})$. Also derive the distribution of the sample correlation coefficient when $\rho = 0$. 10

6. (a) Let the random variables X and Y follow jointly a Bivariate Normal distribution with $(0, 0, 1, 1, \rho)$. Obtain the correlation coefficient between $G(X)$ and $H(Y)$, where $G(X)$ and $H(Y)$ are the *cfds* of X and Y respectively.

P.T.O.

(b) Let $X_1, X_2, \cdots, X_n$ be a random sample of size n drawn from a population with the *cdf* $F(x)$. Derive the expectation of the rth order statistic.

5+5=10