

**M.Sc. Semester-II Examination 2017**  
**Computer Science**  
**Course : MCSC-21**  
**(Design and Analysis of Algorithms)**

**Time : 3 Hours**

**Full Marks : 40**

Questions are of value as indicated in the margin

Answer question number **1** and **any four** from the rest

1. (a) Let  $f(n)$  and  $g(n)$  be asymptotically non-negative functions. Using the basic definition of  $\Theta$  – notation, prove that  $\max(f(n), g(n)) = \Theta(f(n) + g(n))$ .

- (b) Solve the following recurrence relation :

$$T(n) = 2T(\sqrt{n}) + \log(n)$$

- (c) Define all the edges of directed graph after applying DFS algorithm. Give an example to show all the edges.

- (d) Study the following program pseudo-code and compute its time complexity:

$A()$

{int  $n = 2^{2^k}$

for( $i = 1; i \leq n; i++$ )

{ $j = 2;$

while( $j \leq n$ )

$j = j^2;$

}

}

Assume  $k$  to be a positive integer.

4×2=8

2. State and prove the master theorem for the recurrence  $T(n) = aT(n/b) + f(n)$  where  $n$  is an exact power of  $b$  and standard notations have been used.

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3. (a) Let  $A_1, A_2, \dots, A_n$  be  $n$  matrices of order  $p_1 \times p_2, p_2 \times p_3, \dots, p_n \times p_{n+1}$  respectively. Write matrix-chain multiplication algorithm for an optimal parenthesization.

- (b) Let  $B$  be another matrix of order  $p_r \times p_r$  inserted at the  $r^{th}$  position where  $1 \leq r \leq n+1$ . Modify the above algorithm in such a manner that it needs minimum number of computations.

3+5=8

4. (a) Write an algorithm to multiply two polynomials of degree bound  $n$  by divide & conquer approach.

- (b) Multiply two polynomials  $(1 + x + 2x + 3x^2)$  and  $(3 + 2x + x^2 + x^3)$  by using the above algorithm.

4+4=8

5. (a) How would you modify Strassen's algorithm to multiply two  $n \times n$  matrices in which  $n$  is not an exact power of 2? Show that the resulting algorithm runs in time  $\Theta(n^{\log_2 7})$ .

P.T.O.

(2)

- (b) Write an algorithm to find all pair shortest paths in a directed graph.
- (c) What is the role of transitive closure of a graph? 4+2+2=8
6. (a) Describe LU decomposition algorithm. Analyse its time complexity.
- (b) Find an LU decomposition of the matrix
- $$\begin{pmatrix} 4 & -5 & 6 \\ 8 & -6 & 7 \\ 12 & -7 & 12 \end{pmatrix}$$
- (3+2)+3=8
7. (a) What do you mean by P-class and NP-class problems?
- (b) Write a procedure for establishing that a problem is NP-Complete.
- (c) Suppose that a weighted, directed graph  $G = (V, E)$  has a negative weight cycle. Give an efficient algorithm to list the vertices of one such cycle. Prove that your algorithm is correct. 2+2+4=8
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