

B. Sc. (Honours) Semester-II Examination 2017

Statistics (Honours)

Course: BSC-22

(Real Analysis and Numerical Analysis)

Time : Three Hours

Full Marks : 40

Questions are of value as indicated in the margin

Answer **any four** questions

1. (a) State and prove Cantor's theorem on nested intervals. 5
(b) Prove that the sequence defined by $x_{n+1} = x_n(2 - x_n)$, $0 < x_1 < 1$, $x_n > 0, \forall n$ is convergent. 5
2. (a) Let $\{u_n\}$ and $\{v_n\}$ be bounded sequence. Then show that $\overline{\lim} u_n + \overline{\lim} v_n \geq \overline{\lim} (u_n + v_n)$ 5
(b) If $\{x_n\} \rightarrow l, \{y_n\} \rightarrow m$, then show that $\{x_n^2 - x_n y_n\} \rightarrow l^2 - lm$. 5
3. (a) Establish the convergence and find the limit of the sequence $u_n = \left(1 + \frac{1}{3n}\right)^n$. 5
(b) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$. 5
4. Discuss the convergence of the following series by using appropriate test.
(a) $a + b + a^2 + b^2 + a^3 + b^3 + \dots, 0 < a < b < 1$. 5
(b) $1 + \frac{\alpha \cdot \beta}{1 \cdot \gamma} x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1 \cdot 2 \cdot \gamma \cdot (\gamma+1)} x^2 + \frac{\alpha(\alpha+1)(\alpha+1)\beta(\beta+1)(\beta+2)}{1 \cdot 2 \cdot 3 \cdot \gamma \cdot (\gamma+1) \cdot (\gamma+2)} x^3 + \dots$ 5
5. (a) Derive the relation between Δ and E operators. 2
(b) Find the n^{th} order finite difference of $c^{a+bx} + x^n$. 5
(c) Find $\frac{\Delta}{E} \sin 2x$. 3
6. (a) Describe Newton's backward interpolation formula. 5
(b) Using the method of separation of symbols (or, otherwise) show that
$$u_0 - u_1 + u_2 - u_3 + \dots = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \frac{1}{16}\Delta^3 u_3 + \dots$$

Here $u_n = Eu_{n-1}$. 5
7. (a) Derive Simpon's $\frac{1}{3}^{rd}$ and $\frac{3}{8}^{th}$ rule for numerical integration. 6
(b) Describe the convergence of Newton-Raphson method. 4