M.Sc. Examination, 2019

Semester - II

Physics

Paper: MPC-22

(Classical Mechanics-II)

Time: Three Hours **Full Marks: 40**

Questions are of equal value or as indicated in the margin.

Answer ***any four*** questions

1. a) Obtain Hamilton-Jacobi equation for the Hamilton's principal function S and bring out its physical significance. 3
   b) A charge particle is moving in presence of a uniform magnetic field along z-direction.
      i) Formulate the Hamilton Jacobi equation for the system. 2
      ii) Obtain the time evolution of the system. 5

2. a) Define angle variable for a periodic separable system. How do you obtain the time period of the periodic motion from the action angle variables. 3
   b) Suppose a ball of mass m is placed in a constant gravitational field with acceleration due to gravity is g. It is released from a given height h with a null initial speed and it bounces elastically on the ground.
      i) Draw the phase portrait for the system. 1
      ii) Determine the angular frequency for the motion of the system. 4
      iii) Quantize the problem in the semi-classical approach according to Einstein-Brillouin Kramers rule. 2

3. a) Formulate the canonical classical perturbation theory for the Hamiltonian $H = H_0(q, p, t) + \Delta H(q, p, t)$, where $\Delta H(q, p, t)$ can be treated as the perturbation. 4
   b) Obtain the immediate correction of the frequency of oscillation from harmonic approximation. 6

4. a) The Lagrangian density of the system $\mathcal{L} = -\frac{1}{4}\sum_{\mu\nu} F^{\mu\nu}F_{\mu\nu}$, where $F^{\mu\nu}$ is the electromagnetic field tensor ($F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, A^μ being the vector potential for the electromagnetic field). Show that $\frac{\partial \vec{E}}{\partial t} = \vec{\nabla} \times \vec{B}$, where $F^{10} = E_x$, $F^{2,0} = E_y$, $F^{3,0} = E_z$, $F^{3,2} = B_x$, $F^{1,3} = B_y$, $F^{2,1} = B_z$. 7
   b) Obtain the Hamiltonian density of the system. 3

5. a) Show that Maxwell equation is not invariant under Galilean transformation and the velocity of electromagnetic wave is different in different inertial reference frame. 8
   b) Show that a particle with vanishing mass moves with the velocity of light. 2

6. a) Consider a non-inertial frame S', where the reference frame is rotating with an angular velocity ω with respect to an inertial frame. Obtain the metric tensor of this non-inertial frame assuming z-axis of the inertial frame as the axis of rotation. 5
   b) Obtain the equation of motion of a free-particle in the curve-linear space in terms of Affine connection. 3
   c) Show that Affine connection is not a tensor. 2

7. a) Write down Einstein field equation explaining all the quantities involved in the equation. 2

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b) The metric out side a black hole of mass M describing the space-time is given by

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}c^2\right)dt^2 - \frac{1}{1 - \frac{2GM}{c^2 r}}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

where the four dimensional coordinate space is expressed by time coordinate t and three space coordinate in the spherical coordinate system (r, θ, ϕ). Here G is the universal gravitational constant and c is the velocity of light in free space.

i) Find the radius at which a photon can describe a circular orbit. 6

ii) What would a fixed observer at this radius measure as the period of the photon orbit? (Given, Christoffel symbols $\Gamma^0_{01} = \frac{1}{2}\frac{\partial v}{\partial r}$, $\Gamma^3_{13} = \frac{1}{r}$, $\Gamma^3_{23} = \cot(\theta)$, $\Gamma^2_{33} = -\sin(\theta)\cos(\theta)$ and all other $\Gamma^0_{i,j}$, Γ^3_{ij} with $i = 0, 1, 2, 3,\ \ j = 0, 1, 2, 3$ are zero). 2
