M.Sc. Examination, 2019
Semester - II
Physics
Paper: MPC-21
(Mathematical Methods-II)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Answer ***Question No. 1*** and ***any three*** from the rest of the questions.

Q.1 (a) Obtain the Fourier Transform of a wave pulse which is a function of time and represented by $f(t) = \begin{cases} h & \text{for } |t| < T \\ 0, & \text{elsewhere} \end{cases}$. Plot the Fourier Transform, $g(\omega)$, as a function of ω.

(b) If the Laplace transform of $f(t)$ is represented as $\bar{f}(s)$, then show that Laplace transform of $f(at)$ is equal to $\frac{1}{a}\bar{f}(s/a)$ where a is a constant.

(c) How can one solve the following integral equation with the help of Fourier transform?

$$\varphi(x) = f(x) + \lambda \int_{-\infty}^{\infty} K(x-t)\,\varphi(t)\,dt.$$

(d) If you have a sub-Group ***H*** of order h of the Group ***G*** of order $g>h$, then show that g is integral multiple of h.

(e) Consider a Cyclic Group of three elements: **A** and $\mathbf{A}^2$ =**B** and $\mathbf{A}^3$ =**E** (identity Element). Show the classes in the Group. How many irreducible representations are possible?

5×2=10

Q.2 (a) State and prove Convolution theorem of Fourier Transform.

(b) Obtain the Fourier's transform of $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ and thus evaluate $\int_0^{\infty} \frac{\sin(x)}{x}\,dx$.

(c) Define Fourier sine and cosine transforms and prove the following:

i. $F_s\left|\frac{\partial^2 u(x)}{\partial x^2}\right| = ku(0) - kF_s[u(x)]$, $\boldsymbol{F_s}$ denotes the Fourier sine Transform.

ii. $F_c\left|\frac{\partial^2 u(x)}{\partial x^2}\right| = -\left(\frac{\partial u(x)}{\partial x}\right)_{x=0} - k^2F_c[u(x)]$, $\boldsymbol{F_c}$ denotes the Fourier cosine Transform

2+4+(2+2)

Q.3 (a) A particle of mass 'm' can oscillate about the position of equilibrium under the effect of a restoring force mk^2 (k is the spring constant) times the displacement. If started from rest by a constant force F which acts for a time T and then cesses, obtain the displacement of the oscillator at an instant of time t by adopting unit step function and Laplace Transform method.

(b) Using step functions obtain the Laplace transform of the following function:

$$f(t) = \begin{matrix} \sin(t), & 0 \le t < \pi \\ \sin(2t), & \pi \le t < 2\pi \\ \sin(3t), & \ge 2\pi \end{matrix}$$

(c) Find the inverse Laplace Transform of $\bar{f}(s) = \frac{1}{s(s+a)^3}$

4+4+2

P.T.O.

Q.4 (a) Define Fredholm and Volterra integral equations of first and second kind. Give an example in each case.

(b) Solve the following integral equations:

i. $\int_0^x e^{x-t}\, y(t)dt = e^x + x - 1$

ii. $y(x) = \sin(x) + 2\int_0^x \cos(x-t)\, y(t)dt$

iii. $y(x) = \lambda \int_{-1}^{1} (x+t)\, y(t)dt$, where λ is a parameter.

(c) The general second-order linear differential equation with constant coefficients is given as $y(x)'' + a_1 y'(x) + a_2 y(x) = 0$. The boundary conditions are $y(0) = y(1) = 0$. Convert the differential equation into an integral equation. **(2+2+3)+3**

Q.5 (a) State the properties of a Group.

(b) Consider a square geometry and obtain all the symmetry operations corresponding to that geometry. Construct the multiplication table and show that the symmetry operations form a discrete group of order eight.

(c) Identify the Classes in the Group. **2+6+2**

Q.6 (a) State and prove Rearrangement Theorem.

(b) Explain the reducible and irreducible representations of a finite and discrete Group

(c) Define character of an element of a Group and brief its connection with the Classes of the same Group.

(d) Construct the character table for the equilateral triangular symmetry Group. **2+2+2+4**

Q.7 (a) Prove that any matrix representation T of a finite discrete Group whose elements may be non-unitary, is equivalent to a representation by unitary matrices.

(b) Prove that any matrix which commutes with all the matrices of an irreducible representation must be a constant matrix. **5+5**