B.Sc. (Honours) Examination, 2019
Semester-IV
Statistics
Course : BSC-41
(Estimation)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Let $X_1, X_2,\ldots,X_n$ be i.i.d. Bernoulli (p) random variables and $S_n = \sum_{i=1}^{n} X_i$

   i) Show that no unbiased estimator exists for $\frac{1}{p}$.

   ii) Show that $\frac{s_n(n-s_n)}{n(n-1)}$ is unbiased estimator of pq and has a smaller variance than the estimator $\frac{s_n}{n}\left(1-\frac{s_n}{n}\right)$.

   b) Let $X_1, X_2,\ldots,X_n$ be i.i.d. Poisson (λ) variables.

   i) Proposed an unbiased estimator of $e^{-\lambda}$.

   ii) Is it UMVUE? Justify your answer. 3+4+3=10

2. a) "Consistent estimator is unbiased" – is the statement true?

   b) Let t be a consistent estimator for θ and let $\psi(\theta)$ be a continuous fn of θ. Prove that $\psi(t)$ is a consistent estimator of $\psi(\theta)$.

   c) Let $(X_1,X_2,\ldots,X_n)$ be a ramdom sample from a rectangular distribution $f(x,\theta)=\frac{1}{\theta}$, $0<x<\theta$. Show that the geometric mean

$$t(x)=\left(\prod_{i=1}^{n} X_i\right)^{1/n}$$ is a consistent estimator of θe^{-1}. 2+3+5=10

3. a) Let X_i (I = 1,2,…,n) be a ramdom sample from the log normal distribution

$$f(x;\mu,\sigma)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-\{\frac{1}{2\sigma^2}(\log x-\mu)^2\}};x>0$$

   Find a sufficient statistic for (i) μ when σ is known. (ii) σ when μ is known (iii) (μ,σ) both unknown.

P.T.O.

b) Let X be a random variable with p.d.f.

$$f(x;\theta)=\frac{1}{2};\theta<x<\theta+2$$

where θ being real number.

Does there exist any sufficient statistic for θ? Reason your answer. 6+4=10

4. a) What do you mean by most efficient estimator?

b) If t is a most efficient estimator and t' $\dot{a}$ less efficient estimator with efficiency e and the correlation coefficient between t and t' is ρ, show by considering the estimator t'' defined by $(1+e-2\rho\sqrt{e})t''=(1-\rho\sqrt{e})t+(e-\rho\sqrt{e})t'$ that $\rho=\sqrt{e}$.

c) Prove Rao-Gramer inequality. 2+3+5=10

5. a) Let X_1, X_2, …, X_n be a sample from $U\left[\theta-\frac{1}{2},\theta+\frac{1}{2}\right]$. Find the MLE of θ and comment.

b) For a bivariate normal $BN(0,0,\sigma_1^2,\sigma_2^2,\rho)$ deduce the MLE for σ_1^2,σ_2^2 and ρ on the basis of n pairs of sample $(X_1,Y_1)-(X_2,Y_2)\ldots,(X_n,Y_n)$. 4+6=10

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