

M.Sc. Examination, 2019
Semester-II
Statistics
Course : MSC-22
(Applied Multivariate Analysis)

Time : 3 Hours

Full Marks : 40

Questions are of value as indicated in the margin

Answer **any four** questions.
Notations have usual meanings.

1. a) What are principal components? What are the objectives of principal component Analysis? Derive the 1st principal component for a given covariance matrix. 6
- b) Let $\underline{X}' = [\underline{X}_1, \underline{X}_2, \dots, \underline{X}_k]$ have a covariance matrix Σ , with eigenvalue-eigenvector pairs $(\lambda_i, \underline{e}_i)$ $i = 1, 2, \dots, k$, where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k \geq 0$. Let $\underline{Y}_i = \underline{e}_i' \underline{X}$, $i = 1, 2, \dots, k$ be the principal components. Show that

$$\sigma_{11} + \sigma_{22} + \dots + \sigma_{kk} = \sum_{i=1}^k V(X_i) = \sum_{i=1}^k \lambda_i = \sum_{i=1}^k Var(y_i) \quad 4$$

2. a) Write down the orthogonal factor model with clearly stating the assumptions. How does it differ from regression model analysis? 5
- b) Does the solution of Factor model always exist? Illustrate with example. 5
3. a) Explain when do we need to standardize the variables for performing Factor Analysis. Give example. 5
- b) Derive the optimum expected cost of misclassification (ECM) rule for classifying objects into two populations. 5
4. a) Describe hierarchical dustering method. Differentiate the single linkage, complete linkage and average linkage methods. Is there any graphical method to represent hierarchical clustering? If yes then describe it. 7
- b) Distinguish between clustering and classification techniques. 3
5. a) Define canonical variables and canonical correlations. Derive the first pair of the canonical variables and find their canonical correlation for a given covariance matrix.

$$\Sigma = \begin{pmatrix} \Sigma_{11}^{p \times p} & \Sigma_{12}^{p \times q} \\ \Sigma_{21}^{q \times p} & \Sigma_{22}^{q \times q} \end{pmatrix} \quad 6$$

- b) Show that the canonical correlation is invariant under non-singular transformation. 4
6. a) Explain the decision tree method for classification with example. How many types of nodes are there in a decision tree? Write down their names. 5
- b) What is factor rotation? Why do we need that? Explain the rationale behind it. Give one example of transformation matrix used for factor rotation. 5
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