M.Sc. Examination, 2019
Semester-II
Statistics
Course : MSC-21
(Inference-II)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any three** questions from the below. 13×3

1. a) Find the mean and variance of two sample general linear rank statistic.
   b) Show that two sample Wilcoxon rank sum statistic is a special case of general linear rank statistic.
   c) Is two sample Wilcoxon rank sum statistic symmetric around its mean? Justify your answer.
   d) How does two sample Wilcoxon rank statistic relate to Mann Whitney U statistic? 5+2+3+3
2. a) Prove that sign test is Ump test.
   b) For an exponential family with p.d.f. $f_\theta(x) = \alpha(\theta)h(x)e^{c(\theta)T(x)}$ with respect to some measure μ, construct a UMP unbiased test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. Clearly state the necessary conditions.
   c) For a nonparametric test of median being zero under $H_0, let X_\alpha, \alpha = 1(1)n$ be a series of random variables. $R_\alpha^+ s$ are the rank of $|X_\alpha|$. Further let us define an indicator variable Z_α such that
   $$Z_\alpha = 1 \text{ if } X_\alpha > Median$$
   $$0 \text{ if } X_\alpha < Median.$$
   Show that under H_0, R_α^+ and Z_α are independently distributed. 4+6+3
3. a) Establish that Kolmogorov-Smirnov one sample test statistic is distribution free.
   b) What is an $\alpha - similar$ test? Show that $\alpha - similar$ test is unbiased.
   c) Consider Weibull distribution having p.d.f. $c.x^{e-1}c^{-x^c}; X > 0.c > 0$. Do you think that the above family possess MLR property? Purpose a most powerfull test for $H_0 : c = 1$ against $H_1 : c = 2$. 4+4+5
4. a) Define a test having Neyman structure with respect to a sufficient statistic T.
   b) Suppose X and Y be the independent Poisson random variables with parameter λ and μ respectively. Propose a UMP test for $H_0 : \mu \leq \lambda$ against $H_1 : \mu > \lambda$.
   c) Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. Cauchy having p.d.f.
   $$f(x) = \frac{1}{\pi}\frac{1}{1+(x-0)^2}, 0 < X < \infty.$$
   Propose a locally most powerful test for testify $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. 2+5+6

P.T.O.

5. a) Deduce the asymptotic distribution of Kruskal Wallis test statistic, clearly mentioning the basic assumptions.

b) Define Pitman's asymptotic relative efficiency.

c) Let there be a test $H_0 : X \sim f_0$ against $H_1 : X \sim f_1$ when $f_0 = \frac{1}{2}e^{-|x+1|}$ *and* $f_1 = \frac{1}{2}e^{-|x-1|}$

Construct Neyman- Pearson test. 6+2+5