Five Year Integrated M.Sc. Examination, 2019

Semester - VIII

Course: PH-4-8-2A

(Advanced Quantum Mechanics-III)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Answer ***Question No.1*** and ***any three*** from the rest.

1. Answer any **five** from the following: $5 \times 2 = 10$

   (a) If $T^{\mu\nu}$ and $B^{\mu\nu}$ are tensors under Lorentz transformation, prove that $T^{\mu\nu}B_{\mu\nu}$ is a Lorentz scalar.

   (b) Using $S^{\dagger}\gamma^0 = \gamma^0 S^{-1}$ probe that $\bar{\psi}\psi$ transforms as scalar under orthochronous Lorentz transformations where $\bar{\psi} = \psi^{\dagger}\gamma^0$.

   (c) "Schrödinger equation is not a relativistically covariant equation" - Explain.

   (d) Find the solutions of Dirac equation in rest frame.

   (e) Briefly explain the idea of Born approximation.

   (f) What do you mean by scattering state? How dose it differ from bound state?

   (g) Define differential cross section. For what type of potential it would be independent of azimuthal angle ϕ?

2. (a) Starting from the Dirac Hamiltonian in standard form i.e. $H = \vec{\alpha}.\vec{p} + \beta m$, obtain the anticommutation relations among α^1, α^2, α^3 and β. Now define γ matrices and show that $\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}\mathbb{1}$.

   (b) Derive continuity equation for the solutions of Dirac equation. Can you interpret these solutions as wave function in usual sense? Discuss briefly. (4+2)+(3+1)=10

3. (a) Suppose, ψ is a solution of Dirac equation and it transforms as
$$\psi(x) \rightarrow \psi'(x') = S(\Lambda)\psi(x),$$
under Lorentz transformation (LT). Here, Λ is the Lorentz transformation matrix. Find out the constraint on $S(\Lambda)$, for which Dirac equation remains form invariant under LT.

   (b) Suppose for infinitesimal LT, that S matrix can be expressed as $S = \mathbb{1} + \tau$, where τ is infinitesimal matrix. Now, using appropriate representation of infinitesimal LT, prove that
$$\gamma^{\mu}\tau - \tau\gamma^{\mu} = \Delta\omega^{\mu}_{\nu}\gamma^{\nu}.$$
5+5=10

4. (a) Prove the following identity :
$$(\vec{\sigma}.\vec{a})(\vec{\sigma}.\vec{b}) = \vec{a}.\vec{b} + i\vec{\sigma}.(\vec{a} \times \vec{b}),$$
where, σ^i is the i-th component of Pauli matrices which satisfy the following relation : $\sigma^i\sigma^j = \delta_{ij} + i\epsilon^{ijk}\sigma^k$.

   (b) Write the Dirac equation for a charged fermion moving in an electromagnetic field.

   (c) Show that, in non relativistic limit, the above equation reduces to the following form
$$i\hbar\frac{\partial\phi}{\partial t} = \left[\frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 - \frac{e\hbar}{2mc}\vec{\sigma}.\vec{B} + e\Phi\right]\phi,$$
where ϕ is two component Pauli spinor. 3+1+6=10

5. (a) Derive the continuity equation for the solution of Klein-Gordon equation. Is it possible to interpret these solutions as a wave-function in usual sense?

P.T.O.

(b) Let us define γ^5 as follows:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3.$$

Using the anti-commutation relations of γ^μ matrices show that:

i. $\gamma^5\gamma^\mu\gamma^5 = -\gamma^\mu$,

ii. $Tr[\gamma^\mu a_\mu] = 0$,

iii. $Tr[\gamma^\mu a_\mu \gamma^\nu b_\nu] = 4a.b$, where $a.b = a^\mu b_\mu$. (3+1)+(2+2+2)=10

6. (a) Write the time independent Schrödinger equation for a particle with energy E, which is being scattered by a potential $V(r)$. Express the solutions of this equation in terms of Green's function of the Helmholtz type equation.

(b) Calculate the Green's function for this Helmholtz type equation by performing contour equation and show that it is evaluated as

$$G(\vec{r}) = -\frac{e^{ikr}}{4\pi r},$$

where $k = \frac{\sqrt{2mE}}{\hbar}$. Clearly state the reason for choosing the specific type of contour to evaluate this Green's function relevant for scattering problem. Here, you can use the Cauchy's integral formula: $\oint \frac{f(z)}{(z-z_0)}\,dz = 2\pi i f(z_0)$ if z_0 lies inside the contour and the contour is traversed anti-clockwise. (1+3)+6=10