Five Year Integrated M.Sc. Examination, 2019

Semester-VIII

Course: MT-4-8-2B

(Numerics of Partial Differential Equations)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. (a) Consider the second order wave equation $u_{tt} + u_{xx} = 0$. What is the dispersion relation? What do you mean by artificial dissipation? What are the advantages and disadvantages in artificial dissipation?

   (b) Write down carefully the tridiagonal matrix equation that is involved when $u_t = u_{xx}$ is solved by Crank Nicolson scheme. (2+2+2)+4=10

2. (a) Derive continuity equation for fluid flow in differential form. Hence find the continuity equation for incompressible viscous flow.

   (b) Verify whether or not the following difference representation for the continuity equation for a 2-D steady incompressible flow has the conservation property:

$$\frac{v_{i+1,j} - v_{i+1,j-1}}{\Delta y} + \frac{u_{i+1,j} + u_{i+1,j-1} - u_{i,j} - u_{i,j-1}}{2\Delta x} = 0$$

   where u and v are the x and y components of velocity, respectively. (5+1)+4=10

3. (a) Show that a second order wave equation $u_{tt} - c^2 u_{xx} = 0$ is a hyperbolic equation. Describe its domain of dependence and range of influence?

   (b) What is the difference between mathematical domain of dependence and numerical domain of dependence? (3+4)+3=10

4. (a) Derive the following expression and establish its trucation error and its order of accuracy

$$\left(\frac{\partial u}{\partial y}\right)_{i,j} = \frac{1}{12\Delta y}(-25u_{i,j} + 48u_{i,j+1} - 36u_{i,j+2} + 16u_{i,j+3} - 3u_{i,j+4})$$

   (b) What do you mean by forward difference and rearward difference? Show that the scheme $\dfrac{\phi_m^{n+1} - \phi_m^n}{k} + a\dfrac{\phi_{m+1}^n - \phi_m^n}{h}$ is consistent. 5+5=10

5. (a) Consider the function $f(x, y) = e^x + e^y$. Consider the point $(x, y) = (1, 1)$.

   (i) Calculate the exact values of $\dfrac{\partial \phi}{\partial x}$ and $\dfrac{\partial \phi}{\partial y}$.

   (ii) Use first order forward differences, with $\Delta x = \Delta y = 0.1$,calculate approximate values for $\dfrac{\partial \phi}{\partial x}$ and $\dfrac{\partial \phi}{\partial y}$ at (1,1). Calculate the percentage difference when compared with the exact value from part (i).

   (b) The simple implicit scheme applied to the heat equation is given by

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{\alpha}{(\Delta x)^2}(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1})$$

   Determine the stability restrictions (if any) for this algorithm. 5+5=10

P.T.O.

6. (a) Suppose we wish to solve the heat equation with a source term

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + cu$$

using the simple explicit finite-difference method. Determine the stability requirement.

(b) Define wide and compact stencils. Write down the finite difference formula of $\dfrac{\partial^3 u}{\partial x^3}$ in both the wide and compact stencil form for second order accuracy. 5+5=10