Five Year Integrated M.Sc. Examination 2019

Semester-VI

Course: PH-3-6-4(Old)

(Electromagnetic Theory)

Time: Three Hours **Full Marks: 40**

Questions are of equal value or as indicated in the margin.

Answer ***Question No.1*** and ***any three*** questions from the rest.

1. Answer any **five** from the following: $5 \times 2 = 10$
   (a) Define 4-vector and 4-velocity.
   (b) Calculate $U_\mu U^\mu$, where U^μ is 4-velocity.
   (c) Write the transformation rule for the mixed tensor: $A^{\alpha\beta}_{\delta\eta\mu}$.
   (d) Show that the electric and magnetic field remain invariant under guage transformation.
   (e) Write down the Lorentz transformation matrix for a Lorentz boost along z-axis with a velocity $-v$.
   (f) Prove that the metric tensor $g_{\mu\nu}$ transform as an rank two covariant tensor under a coordinate transformation of the form $x^i \to x'^i$.
   (g) Express continuity equation in covariant form.

2. (a) Define covariant vector and contravariant vector with examples.
   (b) Construct Lorentz transformation matrix for a Lorentz boost along $+ve$ x- direction. Now, using this transformation matrix show that the covariant and contravariant vectors follow different transformation rules under Lorentz transformation. 4+(2+4)=10

3. (a) Define event interval. How do we classify the event intervals?
   (b) Show that the charge density (ρ) transforms as a zeroth component of a 4-vector.
   (c) What do you mean by tensor contraction. Show that tensor contraction reduces the tensor rank by 2. (2+1)+5+2=10

4. Electromagnetic field tensor is given by

$$F^{\alpha\beta} = \begin{bmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{bmatrix}$$

   (a) Write the transformation rule for the field tensor under Lorentz transformation along $-ve$ x-direction.
   (b) Suppose an inertial frame S' is moving to the right at the speed v relative to S frame. $F'^{\alpha\beta}$ and $F^{\alpha\beta}$ are the field tensors as measured by the observers standing on the S' and S frames respectively. Hence, find out the relations between the components of electric and magnetic fields in these two frames.

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(c) Show that $\vec{E}.\vec{B}$ is a Lorentz scalar. 1+5+4=10

5. (a) Define 4-potential of an Electromagnetic field.

   (b) Using the 4-potential define field strength tensor ($F_{\mu\nu}$) and obtain the components of the same.

   (c) Show that $F_{\mu\nu}$ is a guage invariant quantity. 2+5+3=10

6. (a) Show that the retarded time equation i.e. $t_r = t - \frac{|\vec{r}-\vec{r'}(t_r)|}{c}$ has only one physically acceptable solution.

   (b) Show that the the retarded 4-potential for a charge q moving with velocity $\vec{v}$ can be expressed as

$$A^{\mu} = \frac{q}{4\pi\epsilon_0 c}\frac{U^{\mu}}{R_{\nu}U^{\nu}},$$

   where, U^{μ} is the 4- velocity and R^{ν} is the four dimensional difference vector between source point and field point. 3+(3+4)=10
