Five Year Integrated M.Sc. Examination, 2019

Semester-VI

Course: MT-3-6-5

(Mathematical Modeling)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. Write down the differential equation of a spring-mass system with no forces other than a spring force and friction. Solve the system and discuss underdamped and overdamped oscillations situation. 10

2. Suppose that the motion of a mass m were governed by the differential equation $m\frac{d^2x}{dt^2} = kx$ where $k > 0$. Show that the phase plane indicates the equilibrium position $(x = 0)$ is unstable. 10

3. Define Lyapunov stability and asymptotic stability of an equilibrium point. Suppose N^* be an equilibrium point of the system: $\frac{dN}{dt} = f(N)$ and $f(N)$ is a continuously differentiable function with $f'(N^*) \neq 0$. Then show that the equilibrium point N^* is asymptotically stable if $f'(N^*) < 0$ and unstable if $f'(N^*) > 0$. (1+1)+8=10

4. Suppose we have a fish population growing logistically and its being harvested with an amount proportional to the stock level. Formulate an appropriate mathematical model with proper justification. Analyze the stability of your proposed model. Also study the dynamics from the view of fishing mortality and discuss sustainable yield. 4+3+3=10

5. Give some real life examples based on fish population, bird population, insect population etc. And discuss the need of discrete-time models. Consider the discrete-time model $N_{t+1} = \alpha N_t$, where N_t is the size of the population at time t and α is the net reproductive rate. Using cobweb analysis, discuss the stability of its equilibrium point. 4+6=10

6. Consider the following predator-prey model

$$\frac{dN}{dT} = rN\left(1 - \frac{N}{K}\right) - cNP$$

$$\frac{dP}{dT} = bNP - mp.$$

Here N, P are the density of the prey and predator population respectively, and r, K, c, b, m are positive parameters. Make suitable transformation to obtain a dimensionless system and hence analyze the stability of various equilibrium points. 4+6=10