Five Year Integrated M.Sc. Examination, 2019

Semester-VI

Course: MT-3-6-4

(Differential Geometry)

Time: Four Hours **Full Marks: 80**

Questions are of value as indicated in the margin.

[All the notations and symbols have their usual meaning]

Answer ***any eight*** questions.

1. a) Define curvilinear coordinates, coordinate surfaces and coordinate lines. Interpret these by taking spherical polar coordinate system. 3+2

   b) Define base vectors and reciprocal base vectors. Deduce their reciprocality relations. 5

2. Define parallelism of a vector field along a curve and in a region. Find a necessary and sufficient condition for parallelism of a vector field. Using parallelism concept find the equation of a straight line in curvilinear coordinate system. 2+3+3+2

3. Deduce Serret-Frenet's formulas related to geometry of a space curve. 10

4. Find the curvature and torsion at any point of the circular helix in cylindrical coordinates

   $$x^1 = a,\ x^2 = \theta,\ x^3 = k\theta,\ a > 0.$$

   Show that the tangent vector λ at every point of the curve makes a constant angle with the direction of X^3-axis. 8+2

5. Obtain the differential equation of geodesic. Show that a curve on a surface is a geodesic iff the tangent vector field of the curve is a parallel vector field along it. 8+2

6. a) Define a geodesic coordinate system. Given a coordinate system x^i, find a geodesic coordinate system $\overline{x}^i$ from x^i with pole at P_0. 1+5

   b) Use geodesic coordinate system to establish Bianchi's identity. 4

7. a) If n_i denotes the normal line to a surface, show that

   i) $n_i = \frac{1}{2} \in^{\alpha\beta} \in_{ijk}\, x^j_\alpha x^k_\beta$

   ii) $b_{\alpha\beta} = g_{ij} x^i_{\alpha,\,\beta}\, n^j$ 3+2

   b) Deduce formulas of Gauss. 5

8. a) Define isometric surfaces. Show that the surfaces

   $$S_1 : y^1 = v^1 \cos v^2,\quad y^2 = v^1 \sin v^2,\quad y^3 = a \cosh^{-1} \frac{v^1}{a}$$

   and

   $$S_2 : y^1 = u^1 \cos u^2,\quad y^2 = u^1 \sin u^2,\quad y^3 = a u^2$$

   are isometric to each other. 1+5

   b) Define Gaussian curvature of a surface. Show that Gaussian curvature is an invariant. 1+3

9. a) Establish $R = -2k$

   where R is Einstein curvature and k is Gaussian curvature of a surface. 4

   b) Calculate the Gaussian curvature for the manifold whose quadratic form is given by $ds^2 = a^2 \sin^2 u^1 \left(du^2\right)^2 + a^2 \left(du^1\right)^2$. 6

P.T.O.

10. What is integrability condition? Establish Weingarten formulas. Derive equations of Gauss and Codazzi from the integrability condition using Weingarten formulas. 2+3+5

11. What is geodesic curvature? State and prove Meusnier's theorem. 2+8

12. a) Show that

$$x_{(1)} + x_{(2)} = 2H$$

$$x_{(1)} x_{(2)} = k$$

where $x_{(1)}$ and $x_{(2)}$ are principal curvatures of a surface, H is mean curvature and k is the Gaussian curvature. Also show that the principal directions are orthogonal to each other. 4+2

b) Show that the normal curvatures in the directions of the coordinate curves are b_{11}/a_{11} and b_{22}/a_{22}. 4