Five Year Integrated M.Sc. Examination, 2019

Semester - VI

Course: MT-3-6-3A

(Mathematics: Numerical Linear Algebra)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. a) Define singular value decomposition. Find singular value decomposition of the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. 5

   b) Write down the algorithm for finding the determinant value of a matrix of order n. Hence find the flop count for the same. 5

2. a) Define symmetric positive definite matrix. Prove that if a matrix of order n is positive definite then it is nonsingular. 1+3

   b) Find LU decomposition of the matrix $A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & -4 & 2 \\ 3 & -6 & -1 \end{pmatrix}$ by Cholesky decomposition method. Hence solve $AX = B$ where $X = [x_1, x_2, x_3]^t$ and $B = [3, -1, -4]^t$. 6

3. a) Show with an example that in partial pivoting the LU decomposition of a matrix A is actually $PA = LU$ where P is a permutation matrix. 5

   b) Derive normal equations of the least square approximation of the third degree polynomial. 5

4. a) Define Householder transformation of a matrix. Show that Householder matrix is symmetric and orthogonal. 2+3

   b) Use house holder matrices to transform $A = \begin{pmatrix} 4 & 1 & -2 \\ 1 & 2 & 0 \\ -2 & 0 & 3 \end{pmatrix}$ to a similar symmetric tridiagonal matrix. 5

5. a) Define induced norm. Show that induced norm is a matrix norm. 5

   b) Show that for all $x, y \in \mathbb{R}^n$, $\|x + y\|_2 \le \|x\|_2 + \|y\|_2$. 5

6. a) Define spectral radius. Find $\|A\|_2$ where $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ using the concept of spectral radius. 2+3

   b) Define condition number. Show that Condition number of A = Condition number of A^{-1}. 2+3

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