Five Year Integrated M.Sc. Examination, 2019

Semester-IV

Course: MT-2-4-2

(Partial Differential Equations)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

[All the notations and symbols are used in their usual forms]

Answer ***any four*** questions.

1. a) Form a partial differential equation by eliminating the function *f*, g from $z = f\left(x^2 - y\right) + g\left(x^2 + y\right)$.
   b) Find the partial differential equation of all spheres of radius λ, having centre in the *xy* plane.
   5+5=10

2. Solve for general solution
   a) $x^2 p + y^2 q = (x + y) z$
   b) $(mz - ny) p + (nx - lz) q = ly - mx.$ 5+5=10

3. Using Charpit method find complete integral of
   a) $xpq + yq^2 = 1$
   b) $z^2\left(p^2 z^2 + q^2\right) = 1.$ 5+5=10

4. a) Find the integral surface of the PDE $x\left(y^2 + z^2\right) p + y\left(x^2 + z^2\right) q = \left(x^2 - y^2\right) z$ which contains the straight line $x + y = 0,\ z = 1.$
   b) If $\alpha_r D + \beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ, then if $\alpha_r \neq 0$, prove that $u_r = \exp\left(-\frac{\gamma_r x}{\alpha_r}\right)\phi_r\left(\beta_r x - \alpha_r y\right)$ is a solution of the equation $F(D, D')z = 0.$ 5+5=10

5. a) State convolution theorem and using it evaluate $L^{-1}\frac{1}{(s+1)\left(s^2+1\right)}.$
   b) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^x.$ 5+5=10

6. a) State and prove the linearity property of Laplace transform.
   b) Find the Laplace transform of
      (i) $\sin^3(2t)$
      (ii) $\left(t^2 + 1\right)^2.$ 4+(3+3)=10