# Five Year Integrated M.Sc. Examination, 2019

**Semester - IV**

**Course: MT-2-4-1 (Old)**

**( Partial Differential Equations )**

**Time: Three Hours** **Full Marks: 60**

Questions are of value as indicated in the margin.

***[All the notations and symbols are used in their usual forms]***

Answer ***Question No. 1*** and ***any four*** from the rest.

1. a) A partial differential equation requires:
(i) exactly one independent variable (ii) two or more independent variables (iii) more than one dependent variable (iv) equal number of dependent and independent variables.

b) Using substitution, which of the following equations are solutions to the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$:
(i) $\cos(3x-y)$ (ii) $x^2+y^2$ (iii) $\sin(3x-3y)$ (iv) $\exp^{-3\pi x}\sin(\pi y)$.

c) Which of the following equations are quasilinear:
(i) $u_{xx}+2uu_{xy}+xu_{yy}-uu_x+u=0$, (ii) $2xyu_{xy}^2+xu_y+yu_x=0$,
(iii) $u_{xx}+uu_{xy}+5u_{yx}+u_{yy}+2u_{yz}+u_{zz}=0$.

d) The partial differential equation $5\frac{\partial^2 z}{\partial x^2}+6\frac{\partial^2 z}{\partial y^2}=xy$: is classified as
(i) elliptic (ii) hyperbolic (iii) parabolic (d) none of these.

e) The following is true for the following partial differential equation used in nonlinear mechanics known as the Korteweg-de Vries equation $\frac{\partial w}{\partial t}+\frac{\partial^3 w}{\partial x^3}-6w\frac{\partial w}{\partial x}=0$:
(i) linear; 3rd order (ii) nonlinear; 3rd order (iii) linear; 1st order (iv) nonlinear; 1st order.

f) Elimination of the number of arbitrary functions determine the order of the PDE
(i) a true statement, (ii) a false statement, (iii) none.

g) The linear PDE of the corresponding algebraic equation $x^2+y^2+(z-c)^2=a^2$ where $a$, $c$ are arbitrary is
(i) $y^2p-xq=0$, (ii) $yp-xq=0$, (iii) $xp-yq=0$, (iv) none.

h) The general solution of the differential equation $x^2p+y^2q=(x+y)z$ is
(i) $z=xyg\left(\frac{x-y}{z}\right)$, (ii) $F\left(\frac{xy}{z},\frac{x-y}{z}\right)$, (iii) $F\left(\frac{xy}{z}\right), G\left(\frac{x-y}{z}\right)$, (iv) none.

i) The region in which the following equation $x^3\frac{\partial^2 u}{\partial x^2}+27\frac{\partial^2 u}{\partial y^2}+3\frac{\partial^2 u}{\partial x\partial y}+5u=0$ acts as an parabolic equation is:
(i) $x>\left(\frac{1}{12}\right)^{1/3}$, (ii) $x<\left(\frac{1}{12}\right)^{1/3}$, (iii) for all the value of $x$, (iv) $x=\left(\frac{1}{12}\right)^{1/3}$.

j) The function $\frac{1}{2}\left(e^t+e^{-t}\right)$ is an
(i) odd, (ii) even, (iii) neither odd or even, (iv) both (i) and (ii). $2\times 10=20$

2. a) Define linear, nonlinear, quasilinear and semilinear PDEs with examples.

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b) Classify the PDE $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x$ as hyperbolic, parabolic or elliptic at every point $(x, y)$ of the domain.

c) Find the characteristic equation, and obtain a canonical form for $u_{xx} + u_{xy} + u_{yy} + u_x = 0$.

4+3+3=10

3. Solve (***any two***):

a) $(x+2z)p + (4zx - y)q = 2x^2 + y$

b) $y^2(x-y)p + x^2(y-x)q = z(x^2 + y^2)$

c) $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$

d) $pq = px + qy$. 5+5=10

4. a) Form a partial differential equation by eliminating the arbitrary function $f$ from the equation $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.

b) Find the integral surface of the linear partial differential equation
$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$
which contains the straight line $x + y = 0, z = 1$. 5+5=10

5. a) State convolution theorem and using it evaluate $L^{-1}\dfrac{1}{(s+1)(s^2+1)}$.

b) Solve $\dfrac{\partial^3 z}{\partial x^3} - 2\dfrac{\partial^3 z}{\partial x^2 \partial y} = 2e^x$. 5+5=10

6. a) State and prove the linearity property of Laplace transform.

b) Find the Laplace transform of

(i) $\sin(ax) + \cos(ax)$

(ii) $(t^2 + 1)^2$. 4+(3+3)=10