Five Year Integrated M.Sc. Examination, 2019

Semester - IV

Course: MT-2-4-1

(Mathematics-IV)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.

Answer ***Question No. 1*** and ***any three*** from the rest.

1. Determine whether each of the following statements is True or False. Justify your answer. (Any five)
   (a) Let A be a square matrix of order 3×3 with $det(A) = 21$, then $det(2A)$ is 168.
   (b) Any 4 linearly independent vectors in R^4 are a basis for R^4.
   (c) If $(G, .)$ is a group such that $a^2 = e, \forall a, \in G$, then G is an abelian group.
   (d) Lines through the origin are not subspaces of $\mathbb{R}^2$.
   (e) If $A = \{1, 2, 3, 4\}$. Let $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)\}$. Then R is not an equivalance relation.
   (f) If one of the eigen values is zero then the matrix is singular. $5\times 3 = 15$

2. (a) Let V be the set of all solutions of a system of m homogeneous linear equations in n variables with real coefficients. Prove or disprove V is a vector space over $\mathbb{R}$ under obvious operations.
   (b) Does the set of all polynomials having exactly of degree n form a vector space? Justify your answer.
   (c) Write down all the subspaces in $\mathbb{R}$, $\mathbb{R}^2$ and $\mathbb{R}^3$. 5+4+6=15

3. (a) Find the dimension and a basis for the following vector space:
   $V = \{(x_1, x_2, x_3) \in R^3/3x_1 - 2x_2 + x_3 = 0 \text{ and } 4x_1 + 5x_2 = 0\}$.
   (b) Find whether v is a linear combination of $v_i's$ where $v = (x_1, x_2, x_3, 1), x_i \in \mathbb{R}$ arbitrary, $v_1 = (1, 2, 3, 0)$, $v_2 = (2, 3, 1, 0)$ and $v_3 = (3, 2, 1, 0)$.
   (c) Show that $\{X, 3X^2, 5 + X\}$ is a basis of P_2 (the set of all polynomials having degree less than or equal to 2). What about $\{2X, X^2-3X, 2X^2\}$? 5+4+6=15

4. (a) Find the eigenvalues and eigenvectors of the matrix A where
   $$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$
   (b) Show that $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 6 & 3 & 7 & 4 & 2 & 1 \end{pmatrix}$ is an odd permutation. Find the order and inverse of α.
   (c) Let $G = \{\begin{pmatrix} a & 0 \\ b & 1 \end{pmatrix} : a, b \in \mathbb{R}, a \neq 0\}$. Show that G becomes a group under usual multiplication. 6+(3+2)+4=15

5. (a) Show that congruence is an equivalence relation.
   (b) Show that the set $\{1, -1, i, -i\}$ forms an abelian group with respect to multiplication. Is this set forms a cyclic group under the same operation. Show that every subgroup of this set forms a cyclic group.
   (c) What is the remainder when 11^{40} is divided by 8.
   (d) Prove or disprove that the binary operation '*' on $\mathbb{N}$ defined by $a * b = a^b \forall a, b \in \mathbb{N}$ is neither commutative nor associative. 3+(3+2+2)+3+2=15

6. (a) If $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, find A^{-1}.

(b) Without expansion show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c})$

(c) Solve the following system of linear equations using matrix method

$$\begin{aligned} x - y + 3z &= 3 \\ 2x - y + 4z &= 3 \\ -x + 2y - 4z &= -4 \end{aligned}$$

5+5+5=15