Five Year Integrated M.Sc. Examination, 2019

Semester-II

Course: MT-1-2-1 (Old)

(Mathematics-II)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin

Answer ***any four*** questions.

1. (a) Find a vector equation and parametric equations for the line that passes through the point (5, 1, 3) and is parallel to the vector $\hat{i} + 4\hat{j} - 2\hat{k}$.
   (b) Find a unit normal vector $\hat{n}$ of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point P(1,0,2).
   (c) Show that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$.
   (d) Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$ 5+3+5+2=15

2. (a) Find a potential function for the vector field, $\vec{F} = 2xy^3z^4\hat{i} + 3x^2y^2z^4\hat{j} + 4x^2y^3z^3\hat{k}$.
   (b) Evaluate $\int_C y^2dx + xdy$, where (i) $C = C_1$ is the line segment from (-5, -3) to (0, 2) and (ii) $C = C_2$ is the arc of the parabola $x = 4 - y^2$ from (-5, -3) to (0, 2). Remark on the different results.
   (c) State Gauss's divergence theorem. Verify this theorem for $F = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$. 4+5+6=15

3. (a) Find the constant "m" such that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{c} = 3\hat{i} + m\hat{j} + 5\hat{k}$ are coplanar.
   (b) Define directional derivative. Find the directional derivative of the function $f(x, y) = xe^{xy} + y$ along the direction of $\theta = \frac{2\pi}{3}$.
   (c) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$, find"t" such that $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$.
   (d) Simplify $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d})$ 3+5+4+3=15

4. (a) Find the tangent plane to the surface with parametric equations $x = u^2, y = v^2, z = u+2v$ at the point (1, 1, 3).
   (b) Define an irrotational vector. Show that the vector function $\vec{F} = (2x - yz)\hat{i} + (2y - zx)\hat{j} + (2z - xy)\hat{k}$ is irrotational.
   (c) Show that the integral $\int_C \vec{F}.\vec{dr} = \int_C (2xdx + 2ydy + 2zdz)$ is independent of path in any domain in space and find its value if C has the initial point A: (0, 0, 0) and terminal point B: (2, 2, 2).
   (d) Evaluate $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x + y)dxdy$ 4+4+4+3=15

5. (a) If the given equation $Mdx + Ndy = 0$ is homogeneous and $(Mx + Ny) \neq 0$, then $\dfrac{1}{Mx + Ny}$ is an integrating factor.
   (b) Find the general and singular solutions of $x^3p^2 + x^2yp + a^3 = 0, \left(p \equiv \dfrac{dy}{dx}\right)$.
   (c) Solve $x^2dx + y^2dy + \dfrac{xdy - ydx}{2(x^2 + y^2)} = 0$. 5+5+5=15

P.T.O.

6. (a) Solve $D^4 - 2D^3 + 5D^2 - 8D + 4)y = e^x$, $\left(D \equiv \frac{d}{dx}\right)$.

(b) Solve the differential equation $\dfrac{d^2y}{dx^2} + 4y = 4e^{(2x)}$ by the method of variation of parameters.

(c) Prove that the equation $x^2(1+x)\dfrac{d^2y}{dx^2} + 2x(2+3x)\dfrac{dy}{dx} + 2(1+3x)y = 0$. is exact and hence find its solution.

5+5+5=15