Five Year Integrated M.Sc. Examination, 2019

Semester-II

Course: MT-1-2-1

(Mathematics-II)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. (a) Find a vector equation and parametric equations for the line that passes through the point (5, 1, 3) and is parallel to the vector $\hat{i} + 4\hat{j} - 2\hat{k}$.
   (b) Find a unit normal vector $\hat{n}$ of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point P(1,0,2).
   (c) Show that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}\, \vec{b}\, \vec{c}]^2$.
   (d) Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$ 5+3+5+2=15

2. (a) Find the constant "m" such that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{c} = 3\hat{i} + m\hat{j} + 5\hat{k}$ are coplanar.
   (b) Define directional derivative. Find the directional derivative of the function $f(x, y) = xe^{xy} + y$ along the direction of $\theta = \frac{2\pi}{3}$.
   (c) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$, find "t" such that $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$.
   (d) Simplify $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d})$ 3+5+4+3=15

3. (a) State Rolle's theorem. Prove that the equation $1 + 2x + x^3 + 4x^5 = 0$ has exactly one real root.
   (b) Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection, and local maxima and minima.
   (c) Verify that $f(x) = 3x^2 + 2x + 5$, $x \in [-1, 1]$ satisfies the hypotheses of Mean Value Theorem. 5+6+4=15

4. (a) If z is a function of x, y where $x = u + v, y = uv$. Prove that
$$\frac{\partial^2 z}{\partial u^2} - 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2} = (x^2 - 4y)\frac{\partial^2 z}{\partial y^2} - 2\frac{\partial z}{\partial y}.$$
   (b) If $u = x^4 y + y^2 z^3$, where $x = rse^t$, $y = rs^2 e^{-t}$ and $z = r^2 s \sin t$, find the value of $\frac{\partial u}{\partial r}, \frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$ when $r = 2, s = 1$ and $t = 0$.
   (c) If $e^{a_1 x_1 + a_2 x_2 + \ldots\ldots + a_n x_n}$, where $a_1^2 + a_2^2 + \ldots\ldots + a_n^2 = 1$. Show that
$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial y_2^2} + \ldots. + \frac{\partial^2 u}{\partial x_n^2} = u$$
5+5+5=15

5. (a) Obtain a reduction formula for $I_{m,n} = \int \sin^m x \cos^n x dx$ where m, n being positive integers, greater than 1, and hence find $\int_0^{\pi/2} \sin^4 x \cos^8 x dx$.
   (b) Show that $\int_0^{\pi/2} \sin^5 x dx = \frac{8}{15}$.
   (c) State the fundamental theorem of Integral calculus. 8+5+2=15

6. (a) State and prove the linearity property of Laplace transform.
   (b) Find the Laplace transform of
      (i) $\cos(at + b)$ where a and b are constants.
      (ii) $e^{at} f(t)$ where a is a constant.
   (c) State convolution theorem and using it evaluate $L^{-1}\frac{1}{(s+1)(s-1)}$. 4+(3+3)+5=15