M.A./M.Sc. Examination 2018
Semester - III
Mathematics
Course: MMO-31 (P2)
(Advanced Topology-I)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any three*** questions from Question Nos. 1 to 5

1. a) Prove that a topological space X is Hausdorff iff limit of a convergent net in X is unique. 4
   b) Let X and Y be topological spaces and $f : X \to Y$ be a function. Prove that f is continuous at x_0 iff whenever a net S converges to x_0, $f \circ S$ converges to $f(x_0)$. 4
   c) Prove that a filter $\Im$ in X is an ultrafilter iff for any $A, B \subset X$, $A \cup B \in \Im \Rightarrow A \in \Im$ or $B \in \Im$. 4

2. a) Prove that in a locally compact Hausdorff space compact neighbourhoods form a local base. 3
   b) Prove that quotient space of a locally connected space is locally connected. 4
   c) Prove that in a space (X, τ) every sequence has a cluster point iff every infinite set has an w-limit point. Hence show that every sequentially compact space is $(B-W)$ compact. 4+1

3. a) Prove that product of a family of topological spaces is compact iff each component space is compact. 1+4
   b) State and prove Embedding Lemma. 1+6

4. a) Let $f : X \to [0,1]$ be a continuous function and for $t \in R$, $F_t = \{x \in X; f(x) < t\}$. Prove that $\{F_t;\, t \in \mathbb{R}\}$ is a family of open sets in X such that $\bar{F}_s \subset F_t$ if $s < t$ and $f(x) = inf\,\{t \in Q;\, x \in F_t\}$. 4
   b) Prove that a topological space X is normal iff for every closed set A in X, every continuous function $f : A \to [-1,1]$ has a continuous extension $F : X \to [-1,1]$. 8

5. a) Describe one-point compactification of a topological space. 5
   b) Prove that one-point compactification of a space is Hausdorff iff the space is locally compact and Hausdorff. Prove that every locally compact Hausdorff space is a Tychonoff space. 5+2

6. Answer ***any two***
   a) Give an example of a T_1 space X and a net in X converging to two distinct points. 2
   b) Prove that a connected T_4 space with at least two points must be uncountable. 2
   c) In a discrete space show that the neighbourhood filters are ultrafilters. 2