

M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMO-31 (A7/P8) (New)

(Lie Theory of Ordinary and Partial Differential Equations)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.

Notations and symbols have their usual meanings.

Answer **Question No.1** and **any three** from rest five.

1. Define Lie group and Lie group of point transformations. Point out the important difference between these two. Show that the set of matrices $\left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}; a, b, c \in \mathbb{R} \right\}$ is a Lie group. 2+2+2+7

Or

Show that $y''(x) + p(x)y'(x) + q(x)y(x) = h(x); p(x), q(x), h(x) \in C(\mathbb{R})$ admits Lie group of point transformations generated by $\hat{X}_1 = \phi_1(x) \frac{\partial}{\partial y}, \hat{X}_2 = \phi_2(x) \frac{\partial}{\partial y}$ where two functions $\phi_1(x)$ and $\phi_2(x)$ are solutions of a differential equation to be determined by you. Use this information to recover the solution (obtained by the method of variation of parameters) of the given equation. 5+8

2. What do you mean by Lie group of point transformations in infinitesimal form and define infinitesimal generator for one-parameter Lie group of point transformations. Find infinitesimal generators for the Lie group of transformations

$$(x, y) \rightarrow (x^*, y^*) = \left(\frac{\epsilon_3 x + \epsilon_4 y + \epsilon_5}{\epsilon_1 x + \epsilon_2 y}, \frac{\epsilon_6 x + \epsilon_7 y + \epsilon_8}{1 + \epsilon_1 x + \epsilon_2 y} \right).$$

Is it possible to get the group of rotations in the (x, y) -plane from the given group. Justify your answer. 1+1+5+2

3. Discuss invariant curve (surface), invariant family of curves (surfaces) and canonical variables involved with the Lie group of point transformations. State the principle for their determination. Find canonical variables for the Lie group of point transformations $(x, y) \rightarrow (x^*, y^*) = (x \cosh \epsilon + y \sinh \epsilon, x \sinh \epsilon + y \cosh \epsilon); \epsilon \in \mathbb{R}$. 1+1+1+2+4

4. State contact conditions relating the differentials $dx, dy, dy_1, \dots, dy_k$ in the geometric space comprising independent and dependent variables x and y respectively. Hence derive formulas for $\eta^{(i)}, i \in \mathbb{N}$ involved in the prolongation of infinitesimal generator $\hat{X} = \xi(x, y) \frac{\partial}{\partial x} + \eta(x, y) \frac{\partial}{\partial y}$ for a Lie group of point transformations. 2+7

P.T.O.

5. Verify whether $y''(x) = \frac{1}{y^3}$ admits Lie group of point transformations generated by $\hat{X}_1 = \frac{\partial}{\partial x}$, $\hat{X}_2 = 2x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}$. Verify whether the Lie algebra $\{\hat{X}_1, \hat{X}_2\}$ is solvable and solve this equation, if yes. 2+2+1+4
6. Find infinitesimal generators for the Lie group of (point) symmetry transformations admitted by the Burger's eq. $u_t + uu_x = u_{xx}$. 9
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