B.Sc. (Honours) Examination 2019
Semester – IV(CBCS)
Mathematics
Course: CC-10
(Probability Theory)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any six*** questions**.**

1. i) Six unbiased dice are thrown simultaneously. Find the probability of getting all six different faces.
   ii) If cards are successively drawn without replacement from a full pack, find the probability that five cards will precede the first ace.
   iii) If two events A and B are independent then show that $\overline{A}$ and $\overline{B}$ are also independent events.
   iv) If a die is thrown n times, show that the probability of an even number of sixes is $\frac{1}{2}\left\{1+\left(\frac{2}{3}\right)^n\right\}$.
   v) Three coins having probabilities of head $1/2, 2/5, 3/7$ respectively are thrown. Find the probability of obtaining exactly one head. 2×5=10

2. i) In a country, 51% of adults are males. One adult is selected at random for a survey. It is learnt that the selected survey person was smoking a cigar. 9.5% of the males smoke cigars, whereas, 1.7% of female smoke cigars. State Bayes' theorem and use it to find the probability that the selected person is a male. 1+4=5
   ii) If $P(A)=0.25$ and $P(B)=0.8$ then show that $0.05 \le P(A \cap B) \le 0.25$. 3
   iii) The probability that A can solve a certain problem is $2/5$ and that B can solve if is $1/3$. If both try it independently, what is the probability that it is solved? 2

3. i) Define distribution function of a random variable. Show that it is continuous on the right at all points. If X is a $N(0,1)$ variate then find the distribution of e^X. 1+1+2=4
   ii) Define probability density function of a random variable. If X is a $\gamma(l)$ variate then find the density function of $\sqrt{X}$. 1+2=3
   iii) If the probability density function of a random variable X is given by
   $$f(x)=\begin{cases} 1-|x|; & |x|<1 \\ 0; & \text{elsewhere,} \end{cases}$$
   then find the variance of X. 3

4. i) Define expectation of a random variable. If r tickets are drawn successively with replacements from an urn containing n tickets numbered $1,2,...,n$, then find the expectation of the greatest number drawn. 1+4=5
   ii) Define median of a continuous distribution. Show that the first absolute moment about any point is minimum when taken about the median. 1+4=5

P.T.O.

5. i) If X and Y are two random variables such that X has the probability density function

$$f(x)=\begin{cases}24x^2; & 0<x<\frac{1}{2}\\ 0; & \text{elsewhere,}\end{cases}$$

and the conditional density of Y, given $X=x$ is

$$g(y/x)=\begin{cases}\frac{y}{2x^2}; & 0<y<2x\\ 0; & \text{elsewhere.}\end{cases}$$

Find the conditional density of X, given $Y=y$ over an appropriate domain to be determined by you. 5

ii) The joint density function of two random varibles X and Y is given by

$$f(x,y)=\begin{cases}x+y; & 0<x<1,\ 0<y<1\\ 0; & \text{elsewhere.}\end{cases}$$

Find the distribution of $X+Y$. 5

6. a) Define covariance and correlation co-efficient of two random variables X and Y. Show that $-1\le\rho(X,Y)\le 1$. 1+1+3=5

b) The joint probability density function of X and Y is given by

$$f(x,y)=\begin{cases}x+y; & 0<x<1,\ 0<y<1\\ 0; & \text{elsewhere.}\end{cases}$$

Compute $\rho(X,Y)$ and write down the regression lines. 5

7. i) If X is a $\gamma\left(\frac{1}{2}n\right)$ variate then show that Y=2X has a $\chi^2(n)$— distribution. 2

ii) If X is a standard normal variate, Y is a $\chi^2(n)$ variate and are independent of each other then show that $Z=\dfrac{X}{\sqrt{Y/n}}$ has t-distribution with n degrees of freedom. 5

iii) If $X=\chi^2(m)$ and $Y=\chi^2(n)$ are two independent χ^2— variates then show that $Z=\dfrac{X/m}{Y/n}$ is a $F(m,n)$ variate. 3

8. i) Show by Tchebycheff's inequality that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least $1/20$. 3

ii) If a random variable X has probability density function

$$f(x)=12x^2(1-x);\ 0<x<1,$$

then compute $P(|X-m|\ge 2\sigma)$ and compare it with the limit given by Tchebycheff inequality. 2+2=4

iii) Obtain Bernoulli's theorem as a particular case of law of large numbers for equal components. 3

9. i) Define convergence in probability. Hence or otherwise establish Bernoulli's theorem on convergence in probability. 1+3=4

ii) Define convergence in distribution. State the Central Limit theorem on convergence in distribution.

Estimate an approximate value of

$$P(-0.3\le Y\le 1.5)$$

Using Central Limit theorem where

$$Y=X_1+X_2+\dots+X_{15}$$

is the sum of a random sample of size 15 from a distribution whose density function is given by

$$f(x)=\begin{cases}\frac{3}{2}x^2; & -1<x<1,\\ 0; & \text{elsewhere.}\end{cases}$$

It is given that

$$\Phi(z\le .50)=0.6915,$$

$$\Phi(z\le .10)=0.5398.$$

1+5=6