B.Sc. (Honours) Examination, 2019

Semester-IV

Physics (Honours)

Course: BPC-41

(Classical Mechanics)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. a) What do you mean by generalized coordinates? What is the advantage of using them?
   b) Consider a pendulum of length *l* having a mass m attached at one end. The other end is fixed such that the pendulum moves in the xy plane. Obtain the Lagrangian for the system and hence obtain the Euler-Lagrange's equations of motion.
   c) Show that the kinetic energy of a system of particles can be expressed as the sum of kinetic energy of motion of the centre of mass and kinetic energy of motion about the centre of mass (1+1)+4+4

2. a) Show that the motion of a two body system under central force can be reduced to an equivalent one body problem.
   b) State Hamilton's variational principle. Hence obtain Euler-Lagrange's equations of motion from it.
   c) Show that the motion of a particle under central force is confined to a plane. 4+(1+3)+2

3. a) Show that the shortest distance between two points is a straight line.
   b) i) For a particle of mass *m* moving in a central potential $-\frac{k}{r}$, show that the quantity $\vec{A} = \vec{p} \times \vec{L} - mk\frac{\vec{r}}{r}$ is conserved, where $\vec{p}$ and $\vec{L}$ are linear and angular momentum respectively.
   ii) Also show that the locus of the particle in the (r, θ) plane is
   $$\frac{1}{r} = \frac{mk}{l^2}\left(1 + \frac{A}{mk}\cos\theta\right)$$ 3+(4+3)

4. a) State and prove Virial theorem.
   b) Show that for an inverse square law of force, $2\overline{T} + \overline{V} = 0$, where $\overline{T}$ and $\overline{V}$ are the time average of kinetic energy and potential energy respectively.
   c) Establish Kepler's 2^{nd} law of planetary motion from the Euler-Lagrange's equations of motion for a particle moving in a central force field. (1+3)+2+4

5. a) Equation of orbit of a particle under the action of central force is given by $r = a(1+\cos\theta)$. Find the law of force.
   b) Obtain Hamilton's canonical equations of motion.
   c) Obtain the general expression for differential scattering cross-section for particles having impact parameter between s and s + ds. 4+3+3

6. a) Consider the potential $V(r) = -\frac{a}{r^2}$, where *a* is a constant. Represent $V_{eff}(r)$ graphically and discuss about the nature of the orbit for $E > 0$.
   b) Obtain the relation between differential scattering cross-sections in L-frame and C-frame.

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c) Determine the total scattering cross-section of a particle scattered from a perfect rigid sphere.
(1+3)+3+3

7. a) The expression for momenta after collision in L-frame are given by

$$\vec{p}_1' = \mu\vec{v} + \frac{m_1}{m_1 + m_2}\left(\vec{p}_1 + \vec{p}_2\right)$$

$$\vec{p}_1' = -\mu\vec{v} + \frac{m_2}{m_1 + m_2}\left(\vec{p}_1 + \vec{p}_2\right)$$

where symbols have their usual meaning.

i) Draw the momentum diagram for the system considering m_2 at rest and $m_1 < m_2$.

ii) From the diagram find out the relationship between angles of scattering in L-frame and C-frame.

b) A mass m moves in a circular orbit of radius r_0 under the potential $-\frac{k}{r^n}$. Show that the circular orbit is stable under small oscillations if $n < 2$.

c) What do you mean by isotropy in space?
(3+2)+3+2