B.Sc. (Honours) Examination, 2019

Semester-IV (CBCS)

Mathematics

Skill Enhancement Compulsory Course: SECC-2

(Tensor Calculus)

Time: Two Hours **Full Marks: 25**

Questions are of equal value.
Notations and symbols have their usual meanings
Answer ***any four*** questions.

1. What do you mean by a scalar, vector and tensor fields. Define covariant, contravariant and mixed tensors, their order and rank. A field $A(p, q, r, s)(\tilde{x})$ depending on the coordinates $\tilde{x}\left(=\left(x^1, x^2, x^3, \ldots\ldots\ldots, x^n\right)\right)$ is such that their relations in two coordinate systems $\tilde{x}$ and $\tilde{\bar{x}}\left(=\left(\bar{x}^1, \bar{x}^2, \ldots\ldots\ldots, \bar{x}^n\right)\right)$ are
$$\bar{A}(p, q, r, s)\left(\tilde{\bar{x}}\right) = \frac{\partial \bar{x}^s}{\partial x^l}\frac{\partial x^k}{\partial \bar{x}^r}\frac{\partial x^i}{\partial \bar{x}^p}\frac{\partial x^j}{\partial \bar{x}^q}\, A(i, j, k, l)(\tilde{x}),\ p, q, r, s = 1, 2, \ldots, n.$$
Determine whether the field is tensor. Determine its order and rank, if year. If $\vec{r}$ be the position vector of a point, determine whether (a) $\frac{\partial \vec{r}}{\partial \bar{x}^p}\left(\equiv \bar{\alpha}(p)\right)$, (b) $d\bar{x}^p\left(\equiv \bar{\beta}(p)\right)$ are tensors. State their order and rank, if yes.

2. Find the covariant and contravariant components of a tensor in (a) spherical polar if its contravariant components in rectangular Cartesian coordinates (x, y, z) are $2xy,\ x+y,\ yz$.
Define symmetric and skew-symmetric tensors. Given A^{ijk} and B_{lmn} are two skew-symmetric tensors. Verify the nature of $C^{ij}_{lm} = A^{ijk}B_{lmk}$.

3. What do you mean by line element of a curve in space? Show that square of the line element can be expressed as a quadratic form. What is the name of the coefficients involved in the quadratic form? Do they behave like a tensor? Give the order and rank, if yes. Provide geometric interpretation of the coefficients. Find the coefficients in the square of the line element described in the cylindrical polar coordinates.

4. Give the parametric representation of a surface and a curve on a surface. Derive the second derivative of position vector of a curve on a surface with respect to its parameter. Is the second derivative of position vector at a point on the surface lie in the tangent plane at that point on the surface? Derive the relation between Christoffel symbol of first, second kind and the metric tensor $g_{\mu\nu}$ of the curved space.

5. Define parallel displacement of a vector on a curved space. If K_ν is the parallel displacement of A_ν at $P(\tilde{x})$ to a neighbouring point $Q(\tilde{x}+d\tilde{x})$. Prove that $dA_\nu = K_\nu - A_\nu = A^\mu y^n_\mu\ y_{n,\nu,\sigma}dx^\sigma = A^\mu \Gamma_{\mu\nu\sigma}dx^\sigma$. Where $\Gamma_{\mu\nu\sigma}$ is Christoffel symbol of first kind.

6. Define curvature tensor. Establish the relation among curvature tensor and the Christoffel symbol of the second kind. Hence or otherwise find the curvature tensor at any point on a spherical surface of constant radius a.

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