Use separate answer scripts for each Unit

B.Sc. (Honours) Examination 2019

Semester – IV(CBCS)

Mathematics

Course: CC-9

(Mechanics-I)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I [Dynamics of a Particle (Marks: 30)]

Answer ***any three*** questions

1. a) Two particles of masses m_1 and m_2 $(m_1 > n_2)$ are connected by a light inextensible string passing over a light smooth pulley, and are allowed to hang freely. Find the equation of the resulting motion, tension of the string and the pressure on the pulley. 5

   b) A particle moves in a straight line OX with an acceleration $\dfrac{\mu}{(\text{distance})^2}$ directed towards the fixed point O on the line. If the particle starts from rest at a point *A* at a distance '*a*' from *O*, show that the period of oscillation is $2\pi\dfrac{a^{3/2}}{\sqrt{2\mu}}$. 5

2. a) Find the expression for the radial and cross-radial component of velocity and acceleration of a particle moving in a plane. 6

   b) A particle of mass *m* moving on a straight line is attracted towards the origin on it with an force $m\mu$ times distance from it and the resistance of the motion at any point is mk times the square of the velocity there. If it starts from rest at a distance '*a*' from the origin, prove that it will come to rest again at a distance *b* from the origin where $(1+2ak)e^{-2ak} = (1-2bk)e^{2bk}$. 4

3. a) Define apse and apsidal distance. Show that at an apse the direction of motion of the particle is perpendicular to the radius vector. 1+2=3

   b) A particle moves under a force $m\mu\{3au^4 - 2(a^2-b^2)u^5\}$, $a > b$ and is projected from an apse at a distance $a+b$ with velocity $\sqrt{\mu}/(a+b)$. Show that the equation of the orbit is $r = a + b\cos\theta$ where $r = \frac{1}{u}$. 7

4. a) What is a simple pendulum? Find its equation of motion. Show that the 2nd approximation of the time period can be obtained as : $T_2 = T_1\left(1+\frac{1}{4}\sin^2\frac{\alpha}{2}\right)$ when $T_1 = 2\pi\sqrt{\frac{\ell}{9}}$, ℓ is the length of the pendulum and α is to be calculated by you. 1+2+4=7

   b) A particle describes a curve, $s = c\tan\psi$ with uniform speed *v*. Find the acceleration indicating its direction. 3

5. a) Define impulse of a force. Show that the impulse of a force is equal to the change of momentum in the direction of force. 1+2=3

   b) A body of mass (m_1+m_2) split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy *E*. Show that if after explosion the parts move in the same line as before, their relative speed is $\sqrt{2E(m_1+m_2)/m_1m_2}$. 4

P.T.O.

5. c) In a simple harmonic motion, the distance of a particle from the middle point of its path in three consecutive seconds are *x, y* and *z*. Show the time of a complete oscillation is $2\pi \Big/ \cos^{-1}\left(\frac{x+z}{2y}\right)$. 3

Unit-II [Statics (Marks: 30)]

Answer ***any three*** questions.

1. a) If $P_i\,(i=1,2,...n)$ be a given system of coplanar forces acting, respectively, at the points $A_i\,(i=1,2,...n)$ of a body, then show that the system can be reduced to a single force *R* at any base point *O* and a couple *G*. 5
   b) Three forces *P, Q, R* act along the sides of a triangle formed by the lines $x+y=3,\ 2x+y=1$ and $x-y=-1$. Find the equation of the line of action of the resultant. 5

2. a) State the principle of virtual work for a single particle under coplanar forces and hence deduce the conditions of equilibrium. 2+3
   b) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ, ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\phi = \frac{3}{8} + \tan\theta$. 5

3. a) Define forces of constraints. When a displacement is said to be consistent with the geometrical condition of the system? 2+3
   b) A regular hexagon is composed of six equal heavy rods freely jointed together, and two opposite angles are connected by a string, which is horizontal, one rod being a contact with a horizontal plane at the middle point of the opposite rod is placed a weight W_1; if W be the weight of each rod, show that the tension of the string is $\frac{3W+W_1}{\sqrt{3}}$. 5

4. a) Let a rigid body be subjected to the three external forces X, Y, Z acting along the axes at *O* with the resultant *R* and a couple *G* of moment $\sqrt{L^2+M^2+N^2}$. Prove that *R* is an invariant but not *G*. 5
   b) Six forces, each equal to *P*, act along the edge of a cube, taken in order, which do not meet a given diagonal. Show that their resultant is a couple of moment $2\sqrt{3}\,Pa$, where *a* is the edge of the cube. 5

5. a) Explain what is meant by Poinsot's central axis for a system of forces in three dimensions acting on a rigid body. 5
   b) Two forces *2P* and *P* act along the lines whose equations are $y = x\tan\alpha,\ z = c$ and $y = -x\tan\alpha,\ z = -c$ respectively. Find the equation of the central axis. 5

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