Use separate answer scripts for each Unit

B.Sc.(Honours) Examination, 2019
Semester-II (CBCS)
Mathematics (Honours)
Core Course: CC-4
(Geometry and Vector Calculus)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I [Geometry (Marks-30)]
Answer ***Question No. 1*** and ***any two*** from the rest.

1. Answer ***any five*** questions: 5x2

a) Find the angle through which the axes are to be rotated so that the equation $\sqrt{3}x + y + 6 = 0$ may be reduced to the form $x = c$. Also determine the value of *c*.

b) Find the points on the conic $\frac{4}{r} = 1 - \frac{\sqrt{2}}{3}\cos\theta$, where the radius vector is 6.

c) Find the polar equation of the circle, when the initial line touches the circle at the pole.

d) If PSP' be the focal chord of a conic, then show that $\frac{1}{SP} + \frac{1}{SP'} =$ constant.

e) Find the equation of the sphere, for which the circle
$$x^2 + y^2 + z^2 + 2x - 4y + 5 = 0,\ x - 2y + 3z + 1 = 0$$
is a great circle.

f) Find the values of *k* for which the plane $x + y + z = k$, is a tangent plane to the conicoid $x^2 + 2y^2 + 3z^2 = 36$. Also find the coordinates of the point of contact.

g) Find the equation of the right circular cone, whose vertex is origin, axis is z-axis and which passes through the point (3, –4, 6).

2. a) If by a rotation of axes, the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then show that

(i) $a' + b' = a + b$ and (ii) $a'b' - h'^2 = ab - h^2$. 3

b) Reduce the equation
$9x^2 - 6xy + y^2 - 14x - 2y + 12 = 0$ to its canonical form and determine the type of the conic represented by it. Obtain the coordinates of the vertex and focus. 3+1+1

c) The origin is shifted to the point (–1, 2) and the axes rotated through an angle $\tan^{-1}\left(\frac{5}{12}\right)$. If the coordinates of a point in the new system be (13, –13), then find the coordinates of the point in the old system. 2

3. a) If the sum of the ordinates of two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be *b*, then show that the locus of the pole of the chord which joins them is $b^2x^2 + a^2y^2 = 2a^2by$. 4

b) Define 'Director Circle' of a conic. Show that the equation of the director circle of the conic $\frac{l}{r} = 1 + e\cos\theta$ is $r^2\left(1 - e^2\right) + 2elr\cos\theta - 2l^2 = 0$. Discuss the case, if the conic is parabola. 1+4+1

P.T.O.

4. a) A sphere of constant radius r passes through the origin and cuts the axes in A, B, C respectively. Prove that the locus of the foot of the perpendicular drawn from the origin to the plane ABC is $\left(x^2+y^2+z^2\right)\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)=4r^2$. 4

   b) Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in two perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 3

   c) Show that the feet of the normals from the point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the surface $\frac{\alpha a^2\left(b^2-c^2\right)}{x}+\frac{\beta b^2\left(c^2-a^1\right)}{y}+\frac{\gamma c^2\left(a^2-b^2\right)}{z}=0$. 3

5. a) Find the locus of a luminous point of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ that cuts a parabolic shadow on the plane $z=0$. 3

   b) Show that no two generators of the same system of hyperboloid of one sheet intersect. 2

   c) Show that the generators of the hyperboloid $x^2+y^2-z^2=1$, which intersect on the plane $z=0$, are at right angles. 3

   d) Find the equations to the generating lines of the hyperboloid of one sheet $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$ which pass through the point (2, 3, –4). 2

Unit-II [Vector Calculus (Marks-30)]

Answer ***any five*** questions.

1. a) Given two vectors $\vec{\alpha}=3\hat{i}-\hat{j}$, $\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$, where $\hat{i}, \hat{j}, \hat{k}$ have their usual meanings, express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. 3

   b) Prove $\left(\vec{b}\times\vec{c}\right)\times\left(\vec{a}\times\vec{d}\right)+\left(\vec{c}\times\vec{a}\right)\times\left(\vec{b}\times\vec{d}\right)+\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=-2\left[\vec{a}\vec{b}\vec{c}\right]\vec{d}$. 3

2. a) Find the perpendicular distance from a point $P(\vec{p})$ to the line $(\vec{r}-\vec{a})\times\hat{e}=\vec{0}$. Also find the position vector of the foot of the perpendicular. 2+1

   b) Find the condition that the lines $\vec{r}=\vec{a}+t\left(\vec{b}\times\vec{c}\right), \vec{r}=\vec{b}+p\left(\vec{c}\times\vec{a}\right)$ may intersect. Accepting that condition, find the point of intersection; $\vec{a}, \vec{b}, \vec{c}$ are any three non-coplanar vectors. 1+2

3. Find the volume of the tetrahedron formed by planes whose equations are $y+z=0$, $z+x=0$, $x+y=0$, $x+y+z=1$, using vector method. 6

4. a) Prove $\vec{\nabla}\times\left(\varphi\vec{A}\right)=\left(\vec{\nabla}\varphi\right)\times\vec{A}+\varphi\left(\vec{\nabla}\times\vec{A}\right)$. 3

   b) Evaluate $\vec{\nabla}\cdot\left(\vec{A}\times\vec{r}\right)$ if $\vec{\nabla}\times\vec{A}=\vec{0}$. 3

5. a) Find equations for the tangent plane and normal line to the surface $z=x^2+y^2$ at the point (2, –1, 5) using vector method. 2+2

   b) Find the directional derivative of $\varphi=4xz^3-3x^2y^2z$ at (2, –1, 2) in the direction $2\hat{i}-3\hat{j}+6\hat{k}$. 2

6. Prove the Frenet-Serret formulae. 6

7. Evaluate $\int_c \vec{A}\cdot d\vec{r}$ along the curve $x^2+y^2=1$, $z=1$ in the positive direction from (0, 1, 1) to (1, 0, 1) if $\vec{A}=(yz+2x)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}$. 6

8. Verify the divergence theorem for $\vec{A}=2x^2y\hat{i}-y^2\hat{j}+4xz^2\hat{k}$ taken over the region in the first octant bounded by $y^2+z^2=9$ and $x=2$. 6
