Use separate answer scripts for each Unit

B.Sc.(Honours) Examination, 2019
Semester-II (CBCS)
Mathematics (Honours)
Core Course: CC-3
(Analysis-II and Algebra-II)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I [Analysis-II) (Marks-30)]

Answer ***Question No. 1*** and ***any four*** from the rest.

1. Answer ***any five*** questions: 5x2
   a) Find the volume and surface area of a sphere of radius a.
   b) Show that arbitrary intersection of open sets may not be open and arbitrary union of closed sets may not be closed.
   c) Define the closure of a set. Find closure of the sets
   (i) $Q\bigcup\{1, 2, ..., 100\}$ (ii) $Q\bigcap\{1, 2, ..., 100\}$.
   d) Find the area between the curves $y=\sin x$ and $y=\cos x$ in $[0, 2\pi]$.
   e) Give an example of a sequence $\{x_n\}$ such that
   $\liminf\limits_{n\to\infty} x_n = 0$ and $\limsup\limits_{n\to\infty} x_n = \infty$.
   f) State Taylor's theorem with Lagrange's form of remainder. Also show that it is a generalization of Lagrange's mean value theorem.
   g) Find the envelope of the family of straight lines
   $y = mx + \sqrt{a^2m^2 + b^2}$ (m is parameter).

2. a) Find the necessary conditions for a power series representation of a function. 3
   b) Define upper limit of a sequence $\{x_n\}$. 2

3. a) Expand $\cos x$ as a power series with proper justification. 3
   b) Define characteristic points and envelope of a curve. 2

4. a) If ρ_1 and ρ_2 be radii of curvatures at the ends P and D of conjugate diameters CP and CD respectively on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=I$, where C being the centre of the ellipse, then show that $\rho_1^{2/3}+\rho_2^{2/3}=\frac{a^2+b^2}{(ab)^{2/3}}$. 2.5
   b) For the cycloid $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$, show that $s^2+\rho^2=16a^2$. 2.5

5. State and prove Nested Interval Theorem. 1+4

6. Define a compact set in $\mathbb{R}$. If $K\subset\mathbb{R}$ is compact then prove that it is closed and bounded. 1+4

7. a) Suppose x_0 is a limit point of a set $S\subset\mathbb{R}$. Show that there is a sequence $x_n\in S-\{x_0\}$ such that $x_n\to x_0$. 2
   b) Show that a set is closed if and only if it contains all its limit points. 3

P.T.O.

Unit-II [Algebra-II (Marks-30)]

Answer ***all*** the questions.

1. Answer ***any two*** questions: $2\times5=10$

a) i) Show that the sum off the squares of two odd integers cannot be perfect square. Does there exist any perfect square in the sequence of integers:
11, 111, 1111, ……………………………………? 2+1

ii) Let a and b be two integers with $b \neq 0$. Then show that there exist unique integers r and r that satisfy $a = bq + r$, where $-\frac{1}{2}|b| < r \leq \frac{1}{2}|b|$. 2

b) i) Use Euclidean Algorithm to obtain integers x and y satisfying gcd(119, 272) = $119x + 272y$. 3

ii) For $n \geq 1$, and positive integers a, b, show that the relation $a^n \mid b^n$ implies that $a|b$. 2

c) i) If $p \neq 5$ is an odd prime, prove that either $p^2 - 1$ or $p^2 + 1$ is divisible by 10. 1

ii) How many primes are there upto $2^{2^{99}}$? 1

iii) Obtain three consecutive integers, each having a factor of perfect square. 3

d) i) If $n > 1$ is a composite number, then show that $\varphi(n) \leq n - \sqrt{n}$. 2.5

ii) Find the unit digit of 3^{100} by means of Euler's theorem. 2.5

2. Answer ***any two*** questions: $2\times10=20$

a) i) Let $\rho = \{(a, b) \in \mathbb{Z}\times\mathbb{Z} : 3a + 4b = 7n, \text{ for some } n \in \mathbb{Z}\}$. Show that ρ is an equivalence relation on $\mathbb{Z}$. 2

ii) Show that $\mathbb{R}$ is equipotent with the set $S = \{x \in \mathbb{R} : 0 < x < 1\}$. 3

iii) Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x^2 - 5$ and $g(x) = \frac{x}{x^2+1}$. Find $f \circ g$, $g \circ f$. Also show that $g^{-1}(g(\{2, 3\})) \neq \{2, 3\}$. 1.5+1.5+2

b) i) Suppose a group G contains elements a and b such that $0(a) = 4$, $0(b) = 2$, $a^3b = ba$. Find $0(ab)$. 2

ii) Find all subgroups of the group $\mathbb{Z}$ of all integers under usual addition. 3

iii) In the group S_3, show that the subset $H = \{a \in S_3 : 0(a) | 2\}$ is not a subgroup. 2

iv) Does there exist a subset H of $(Q, +)$ such that $Q = \langle H \rangle$? 2

v) In the group $GL(2, \mathbb{R})$, find two elements A and B of finite order but order of AB is infinite. 1

c) i) Show that the eighth roots of unity form a cyclic group. Find all the generators of this group. 3+1

ii) Let G be a cyclic group of order 42. Does there exist an element of order 6? Justify your answer. 3

iii) Show that $(\mathbb{R}, +)$ is not a cyclic group. 3