# B.Sc.(Honours) Examination, 2018

**Semester - III**

**Physics (Honours)**

**Paper: BPC-31**

**( Mathematical Methods-II )**

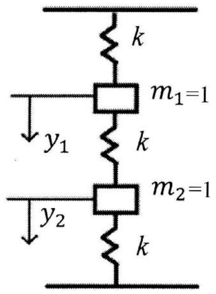
**Time: Three Hours** **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions

**Q.1** **(a)** Consider a Matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and prove that $e^M = I + \left(\frac{e^3-1}{3}\right) M$.

**(b)** Consider a coupled mass-spring system consisting of two unit masses and three identical springs with spring constant $k$ each (shown in the figure). Two ends of the system are fixed. Let the displacements of the masses $m_1$ and $m_2$ at any instant of time are $y_1$ and $y_2$ respectively. Construct the governing equations and solve for $y_1(t)$ and $y_2(t)$ by adopting the methods of solving eigenvalue equations. The initial conditions are $y_1(0) = y_2(0) = 1,\ \dot{y}_1(0) = \sqrt{3k}, \dot{y}_2(0) = -\sqrt{3k}$.




**(c)** Prove that the trace of a square matrix is the sum of eigenvalues and its determinant is the product of the eigenvalues. **2+6+2**

**Q.2** **(a)** Suppose that the set V is set of all real positive numbers (x>0) with addition and scalar multiplication defined as follows:

$x + y = xy$ and $cx = x^c$ where C is a scalar. Show that this set under such addition and scalar multiplication rules form a vector space.

(b) Let W be a set of all points $(1, x_1, x_2)$ in $\mathbb{R}^3$. Is this a subspace of $\mathbb{R}^3$?

(c) Let W be the set of all polynomials of degree exactly $n$. Is this a subspace of F$[a,b]$, a set of all real valued functions on the interval $[a,\ b]$. **6+2+2**

**Q.3** **(a)** Determine if the following sets of vectors are linearly dependent or independent:

(i) $p_1 = x - 3, p_2 = x^2 + 2x,$ and $p_3 = x^2 + 1$ in $P_2$ *i.e.*, polynomials of second order.

(ii) $A = \begin{pmatrix} 8 & -2 \\ 10 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} -12 & 3 \\ -15 & 0 \end{pmatrix}$, the square matrices of order 2.

**(b)** Determine if each of the set of vectors will be a basis for $\mathbb{R}^3$:

(i) $u_1 = (1, -1, 1), u_2 = (0, 1, 2)$ and $u_3 = (3, 0, -1)$

(ii) $u_1 = (1, 1, 0),$ and $u_2 = (-1, 0, 0)$.

**(c)** Determine a basis and dimension for the null space of the matrix A.

$$A = \begin{pmatrix} 7 & 2 & -2 & -4 & 3 \\ -3 & -3 & 0 & 2 & 1 \\ 4 & -1 & -8 & 0 & 20 \end{pmatrix}.$$

**2+2+6**

**P.T.O.**

**Q.4** **(a)** Locate the singularities along with their nature of the function $f(z) = \frac{\sqrt{z}}{z^2+2z+2}$ in a complex plane.

**(b)** Does the following limit exist? Justify.

$$\lim_{z\to 0} \frac{\mathrm{Re}(z^2)+\mathrm{Im}(z^2)}{z^2}$$

**(c)** Find the radius of convergence of the following series:

(i) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$

(ii) $\sum_{n=0}^{\infty} n!\, z^n$

(iii) $\sum_{n=0}^{\infty} (z+5i)^{2n}(n+1)^2$

(iv) $\sum_{n=0}^{\infty} 2^n (z-1)^n$

**(d)** Can $u(x,y) = 2y - y^2 + x^2 + 2x$ be a real part of an analytic function in a complex plane? **2+3+4+1**

**Q.5** **(a)** **(i)** Box-I contains 3 red and 2 blue balls while box-II contains 2 red and 8 blue balls. A fair coin is tossed. If the coin turns up head, a ball is drawn from box-I else a ball is drawn from box-II. What is the probability that a red ball is drawn?

**(ii)** If in the problem **(i)** the toss of the coin is not revealed, but it is said that a red marble is drawn, then what is the probability that the box-I was chosen?

**(b)** The probabilities that a husband and a wife will be alive 20 years from now are given by 0.8 and 0.9 respectively. Find the probabilities that in 20 years both, neither and at least one will be alive.

**(c)** Consider three particles A, B and C, each with spin that can take values $\pm 1$. Let $S_A = 1$, $S_B = 1$ and $S_C = -1$ at a given instant. In the next instant, each S value can change to –S with probability $\frac{1}{3}$. Find the probability that $S_A + S_B + S_C$ remain unchanged.

**(d)** Calculate the average and the variance for a Poisson's distribution. **(1+2)+2+2+3**

**Q.6** **(a)** Expand $f(z) = \frac{1+z}{1-z}$ about $z = i$ and find the region of validity.

**(b)** Expand $f(z) = \frac{z+1}{z(z-4)^3}$ about $z = 4$ in the domain $0 < |z-4| < 4$.

**(c)** Evaluate $\frac{1}{2\pi i} \oint \frac{e^z}{z-2} dz$ over the circles $|z| = 3$ and $|z| = 1$.

**(d)** Evaluate $\oint \frac{e^{3z}}{z - i\pi} dz$ over the elliptical path described by $|z-2| + |z+2| = 6$. **2+2+3+3**

**Q.7** **(a)** State and prove Jordan's Lemma.

**(b)** State Cauchy's Residue theorem in a complex plane.

**(c)** Applying Cauchy's Residue theorem, evaluate the following:

(i) $\int_0^{\infty} \sin(x^2) dx$

(ii) $\int_0^{\infty} \cos(x^2) dx$. **4+1+5**