

M.A./M.Sc. Examination 2019
Semester - IV
Mathematics
Optional Course: MMO-41(P05) (New Syllabus)
(Algebraic Coding Theory-II)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.
 Notations and symbols have their usual meanings.
 Answer **any four** questions.

1. a) Show that $\Pi: F_q^n \rightarrow F_q[x]/\langle x^n - 1 \rangle$ is an F_q -linear transformation of vector spaces over F_q .
 Prove that a nonempty subset C of F_q^n is a cyclic code if and only if $\Pi(C)$ is an ideal of $F_q[x]/\langle x^n - 1 \rangle$. 2+5
- b) Find the generator polynomial of the cyclic code $\{000, 110, 011, 101\}$ in F_2^3 . 3
2. a) Find generator matrix and parity check matrix of a binary $[7, 4]$ -cyclic code with generator polynomial $1 + x^2 + x^3$. 5
- b) Let $g(x)$ be the generator polynomial of a q -ary $[h, k]$ -cyclic code C . Let $h(x) = \frac{(x^n - 1)}{g(x)}$. Then prove that $h_0^{-1}h_R(x)$ is the generator polynomial of $C^\perp$, where h_0 is the constant term of $h(x)$. 5
3. a) Show that a q -ary BCH code of length $q^m - 1$ with designed distance δ has dimension at least $q^m - 1 - m(\delta - 1)$. Find the dimension of a narrow-sense BCH code of length 63 with designed distance 5. 5+2
- b) Show that two narrow-sense binary BCH codes, each of its length $2^m - 1$ with designed distances $2t$ and $2t + 1 (t \geq 1)$ respectively are same. 3
4. a) Let C be a q -ary RS-code generated by $g(x) = \prod_{i=1}^{\delta-1} (x - \alpha^i)$ with $2 \leq \delta \leq q - 1$. Show that extended code $\bar{C}$ is a MDS-code. 5
- b) Consider a 7-ary RS-code of length 6 with generator polynomial $(x - 3)(x - 3^2)(x - 3^3)$. Find a $[7, 3, 5]$ MDS - code from the given code. 5
5. a) Let α be a root of $1 + x + x^3 \in F_2[x]$. Then find the polynomials $\prod_{r \in Q_7} (x - \alpha^r)$ and $\prod_{n \in N_7} (x - \alpha^n)$. 5
- b) Define r -singular point of a bivariate polynomial $Q(x, y) \in F_q[x, y]$. If $(\alpha, \beta) \in F_q^2$ is an r -singular point of $Q(x, y) \in F_q[x, y]$, then show that $(x - \alpha)^r \mid Q(x, f(x))$, for any $f(x) \in F_q[x]$ with $f(\alpha) = \beta$. 2+3

P.T.O.

6. a) Let α be a primitive element of the finite field F_q , and let $q-1 \geq \delta \geq 2$. Show that the narrow-sense q -ary RS-code with generator polynomial $(x-\alpha)(x-\alpha^2) \dots (x-\alpha^{\delta-1})$ is equal to $\{(f(1), f(\alpha), f(\alpha^2), \dots, f(\alpha^{q-2})) : f(x) \in F_q[x] \text{ and } \deg f(x) < q-\delta\}$. 5
- b) Define Goppa code. Find a necessary and sufficient condition for a codeword to be a Goppa code. 1+4
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