M.A./M.Sc. Examination, 2019

Semester-IV

Mathematics

Optional Course: MMO-41(A13/P11) (New Syllabus)

(Weak Formulation of Elliptic Partial Differential Equation)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Symbols have their usual meaning.

Answer ***any four*** questions.

1. a) Define test functions and distributions over the space of test functions. State some properties frequently used in their exercise. Cite fundamental difference between the regular and singular distributions. Hence conclude whether distributions may be regarded as test functions. Justify your answer. 1+1+1+1+1

   b) Define δ-distribution. Find the solution of the equation $(x-\xi)t(x-\xi)=g(x)$. 1+4

2. a) Define the symbol $P\left(\frac{1}{x}\right)$ and verify whether this may be regarded as a distribution over a suitable space of test functions to be determined by you. 5

   b) Establish the formula $\lim_{\epsilon\to 0}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(x)}{x\pm i\varepsilon}dx=\pm\frac{1}{2}\langle\delta,\varphi\rangle+\frac{1}{2\pi i}\left\langle P\left(\frac{1}{x}\right),\varphi\right\rangle$. Hence write down $\frac{1}{x\pm i0}$ in terms of $\delta(x)$ and $P\left(\frac{1}{x}\right)$. 3+2

3. a) Define derivative of distribution. Probe that derivative of distribution follows derivation law. Verify whether the function $|x|$ may be regarded as a distribution? Find the first and second distributional derivatives of $|x|$, if yes. 1+1+1+2

   b) Define classical, weak and distributional solution of a differential equation. Find the general solution of the equation $x^p\frac{dt}{dx}=0,\ p\in\mathbb{N}$. State the nature of its solution. 2+3

4. a) Define convolution of two functions. Establish the formula $(D^kS)*t=D^k(S*t)=S*D^kt$ for any two distributions $s,t\in D'$ for which $s*t$ exists. 1+4

   b) Find the fundamental solution involved with the elliptic boundary value problem - $\frac{d^2u}{dx^2}=f(x),\ 0\le x\le 1,\ u(0)=0=u(1)$. Hence find the solution to the problem. 3+2

5. a) Define Hölder and Sobolev spaces. Are Hilbert spaces Sobolev space? Define mollifier and the process of mollification of a locally integrable function. 1+1+1+1+1

   b) Define multiresolution analysis of a Hilbert space and point out fundamental ingredients involved there. Find low- and high-pass filters, wavelets, basis of approximation- and detail- spaces $(V_j$ and $W_j)$ of multiresolution analysis of a Hilbert space generated by the Haar function. 1+1+3

P.T.O.

6. a) Define elliptic differential operator in a domain $\Omega \sqsubset \mathbb{R}^n$ having smooth boundary Γ. What do you mean by a weak solution of a homogeneous and inhomogeneous boundary value problems involving elliptic differential operator with Dirichlet boundary conditions. Explain the strategy for obtaining bilinear operator associated to these problems. 1+2+2

   b) Define trace operator. State the continuity and coercive property of a bilinear operator defined over a Hilbert space. State and prove Lax-Milgram theorem. 1+1+3