M.A./M.Sc. Examination, 2019

Semester-IV

Mathematics

Optional Course: MMO-41(A09) (New Syllabus)

(Nonlinear Differential Equations-II)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and Symbols have their usual meanings.
Answer ***any four*** questions.

1. a) If $y = e^x$ is a solution of $\frac{dy}{dx} = e^{2x} - ye^x + e^{-x}y^2$, find another solution of the ODE using the transformation $y = e^x + \frac{1}{v}$.

   b) Using tanh-coth method solve Burger's equation. 5+5

2. Find the solution of *KdV* equation and using the Conservation laws of *KdV* equation, find the analytical solution of damped forced *KdV* equation. 3+7

3. Using Hirota Direct method, find one soliton solution of the *KdV* equation. 10

4. Derive Lax compatibility equation. Hence find the Lax pair of *KdV* and *MKdV* equation. 3+7

5. Let $y = y_1(x)$ be a particular solution of the *Strum-Liouville* problem

   $$y'' + (\lambda - u)y = 0, \quad (1)$$

   where the value of the spectral parameter $\lambda = \lambda_1$. Consider the *Darboux* transformation $y \to y[1] = \left(\frac{d}{dx} - l\right)y$ of an arbitrary solution of (1) where $l = \frac{d}{dx}(\log y_1)$.

   Show that $y[1]$ obeys the following equation

   $y''[1] + (\lambda - u[1])y[1] = 0$, where $u[1]$ is given by $u[1] = u - \frac{d}{dx}(l) = u - \frac{d^2}{dx^2}\log y_1$. 10

6. Find one soliton solution of *Sine-Gordon* and *Liouville*'s equation by using Backlund transformation. 5+5

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