M.A./M.Sc. Examination, 2019

Semester - IV

Mathematics

Optional Course: MMO-41(A06) (New Syllabus)

(Dynamical Oceanography)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Symbols are having their usual meanings

Answer ***any four*** questions

1. a) Obtain Gibb's general relation for a multicomponent thermodynamic system of ocean water in the form

$$Td\eta = dE + pd\tau - \sum_{j=1}^{n} \mu_j dc_j.$$

Deduce the corresponding relation for two-component sea water. 4+2

b) Derive the following relations in (T, p, s)-variables:

(i) $\Gamma = \frac{T}{C_p}\frac{\partial \tau}{\partial T}$, (ii) $k_\eta = \frac{C_v}{C_p} k_T$. 2+2

2. a) Assuming the sea-water to be a two-component mixture of salt and pure water, derive from the principle of conservation of mass the following two equations:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{q} = 0 \text{ and } \rho \frac{Ds}{Dt} = -\nabla \cdot \vec{I}_s.$$

Explain the significance of the quantities $\vec{I}_s$ and $\vec{I}_w$ in this connection. 6

b) Obtain the boundary conditions at the free ocean surface taking into account the fact that mass exchange process across the surface amounts to a flux '*b*' of pure water in unit time per unit area. 4

3. Assuming sea-water to be a viscous compressible heat conducting fluid, deduce the equation of energy in the form $\frac{\partial}{\partial t}(\rho E_m) = -div \vec{I}_E$. Interpret the different components of $\vec{I}_E$ and $\vec{E}_m$. 8+2

4. From the basic conservation equations of ocean dynamics, deduce the equations for small amplitude wave motion in the ocean after linearization of these equations. Also, deduce the appropriate boundary conditions. 7+3

5. Explain the assumptions on which the Boussinesq approximations of the governing equations of sea water are based. Show that the equation of continuity under this approximation becomes $\nabla \cdot \vec{q} = 0$, where $\vec{q}$ is the fluid velocity. Also, derive the equations of motion, energy and mass diffusion of salt under this approximation. 10

6. Starting from the equations of small-amplitude gravity wave motion in the ocean along with the boundary conditions in spherical coordinate (λ, ϕ, z) by considering earth's rotation into account, show that the problem of determination of frequencies of free oscillations in horizontally unbounded ocean of finite constant depth may be reduced to two independent eigen value problems, namely problem *H* and problem *V*.
Determine approximately the eigenvalues of problem *V*, assuming sea-water to be incompressible and the Väisälä frequency N_0 to be constant. 10