M.A./M.Sc. Examination, 2019
Semester-IV
Mathematics
Optional Course: MMO-41(A04) (New Syllabus)
(Difference Equations in Ecology)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and Symbols have their usual meaning.
Answer any ***ten*** questions.

1. What is Allee effect? Discuss about Strong and weak Allee effects. 4

2. Solve the migration model
$$x_{n+1} - x_n = 5x_n + 2y_n - 3x_n,$$
$$y_{n+1} - y_n = 7y_n - 2y_n + 3x_n.$$ 4

3. What is Jury's stability criterion? Draw the stability region in trace-determinant plane for a two dimensional discrete system. 4

4. Find the equilibrium points and discuss about their stability of the following system with fear effect:
$$x_{n+1} = x_n \exp\left[\frac{a}{1+ky_n} - d - bx_n - cy_n\right],$$
$$y_{n+1} = y_n \exp\left[\mu c x_n - e\right].$$ 4

5. Show that the logistic equation $x_{n+1} = \gamma\, x_n(1-x_n)$ undergoes a period doubling bifurcation at $\gamma = 3$. 4

6. A certain species of rat has a maximum life span of three years. The probability of a rate surviving its first year is 50%, and the probability of surviving its second year is 60%. On average, each rate in its first year produces 0.6 offspring, rats in their second year produce 2.4 offspring and rats in their third year produce 0.5 offspring each.
The number of rats at initially are 55, 32, 25 for first, second and third year respectively.
Write the Leslie matrix and determine the number of rats after the one year. 4

7. What is Neimark-Sacker bifurcation? Prove that the system
$$x_{n+1} = \alpha x_n(1-x_n) - x_n y_n$$
$$y_{n+1} = \frac{1}{\beta} x_n y_n$$
undergoes Neimark-Sacker bifurcation around the interior equilibrium point. 4

8. Using center manifold theorem, find the nature of the equilibrium point of the system
$$u_{n+1} = -u_n + u_n w_n,$$
$$v_{n+1} = -\frac{v_n}{2} + u_n^2,$$
$$w_{n+1} = \frac{w_n}{2} - u_n v_n.$$ 4

P.T.O.

9. What is bistability? Show that the discrete model with Allee effect

$$x_{n+1} = r\left(1-\frac{x_n}{k}\right)\left(\frac{x_n}{k_0}-1\right)$$

exhibits bistability behavior. 4

10. Draw the bistability region for the two dimensional discrete system

$$x_{n+1} = x_n + \left[x_n(1-x_n)(x_n-\theta) - x_n y_n\right],$$

$$y_{n+1} = y_n + \left[cx_n y_n - dy_n\right]$$

in *c*-*d* parametric space. 4

11. Find the indirect *z*-controller $u_{pred}(t)$ for the discrete system

$$x_{n+1} = x_n \exp\left(r\left(1-\frac{x_n}{k}\right) - \lambda y_n\right),$$

$$y_{n+1} = y_n \exp\left(c\lambda y_n - d\right) - u_{pred}(t)\cdot y_n. \quad 4$$

12. Solve the delay difference equation $U_{k+1} = rU_k(1-U_{k-1})$. 4

13. Find the conditions for the existence of the equilibrium points of the system

$$x_{n+1} = x_n \exp\left[\gamma\left(1-\frac{x_n}{k}\right)\left(\frac{x_n}{x_n+\theta}\right) - \frac{my_n}{ax_n^2+bx_n+c}\right],$$

$$y_{n+1} = y_n \exp\left[\frac{\mu m y_n}{ax_n^2+bx_n+c} - d\right]. \quad 4$$

14. Write a MATLAB code to draw the bifurcation diagrams in a single figure for both the populations of the system

$$x_{n+1} = x_n + ax_n\left(1-\frac{x_n}{k}\right) - \frac{px_n y_n}{x_n+y_n+1},$$

$$y_{n+1} = y_n + \frac{cpx_n y_n}{x_n+y_n+1} - m\cdot y_n. \quad 4$$

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